Volume XVII. *December, 1903.* *Number 6.*

THE

PHYSICAL REVIEW.

ON THE MAGNETIC EFFECT OF ELECTRIC CONVECTION.

By Victor Cremieu and Harold Pender.[1]

A DETAILED discussion of the present state of the question of electric convection has been recently given in an article by Cremieu in the December number of the *Journal de Physique*. Contradictory results have been obtained, some seeming to prove the existence of a magnetic field around a moving charged body, others seeming to prove the non-existence of such an effect.

In the same article the conditions necessary for a proper experiment on electric convection have been clearly set forth. We shall recall that the realization of the following conditions is imperative :

1. That the charge is actually carried along — *entrainée* — by the body in motion.

2. That only this charge can act upon the apparatus destined to detect the magnetic affect attributed to its *entrainement*.

3. That, during its *entrainement*, the charge undergoes no variation ; in particular, that the loss by leakage is limited to a very small fraction of the total charge carried.

In the hope of reconciling the contradictory results obtained by the various experimenters on this question of fundamental importance, M. H. Poincaré took the initiative in the fall of 1902 to bring about a collaboration between two of the experimenters whose results have invariably been in contradiction one to the other. In

[1] Communicated to the Société française de Physique séance du 17 Avril, 1903.

accordance with the suggestion of Lord Kelvin, Paris was chosen as the most favorable place for these joint experiments, and M. E. Bouty gladly put at the disposal of the two investigators his laboratory at the Sorbonne.

Accordingly, invited by Messrs. Poincaré and Bouty in the name of the University of Paris, Mr. H. Pender, whose experiments have invariably given positive results, went to Paris in January, 1903, to work in collaboration with M. V. Cremieu, whose experiments have always been of a negative character. The Johns Hopkins University put at Pender's disposal all the apparatus necessary, and obtained from the Carnegie Institution the funds necessary to defray the expenses of the journey. The expenses of the actual experimenting were met by the Institut de France.

We wish here to express to M. H. Poincaré and to M. E. Bouty our gratitude for their initiative and liberality. Through their invaluable aid in advice and criticism, we have been able to realize in three months an almost complete program of very delicate experiments. Thanks to the most excellent organization of the laboratory of M. Bouty, we have not once been delayed by lack of apparatus or of any physical means whatever.

Also it gives us great pleasure to express our gratitude for the liberalities of the Carnegie Institution, of the Johns Hopkins University, and of the Institut de France.

Repetition of Pender's Experiments.— Cremieu in 1899–1900,[1] was led to repeat Rowland's experiment, and, for considerations elsewhere developed, he modified considerably the method. Instead of the direct magnetic effect, he studied the electromagnetic induction of a moving charged body. The experiment gave negative results, which persisted during a long series of observations, in spite of many modifications of details suggested by objections from numerous sources.

At the suggestion of Rowland, Pender took up the method of Cremieu, improving it in some details. His experiments were first carried on in the Physical Laboratory of the Johns Hopkins University and later in the country, free from the disturbances of any industrial center. The details of these two series of experiments

[1] Cremieu, Thèse de Paris, Gauthier-Villars, 1901, et loc. cit.

have been published elsewhere.[1] The results were invariably positive, and in the second series the observed and calculated values accorded within less than 5 per cent.

Pender brought with him to Paris all his apparatus, which we at once set up under conditions as nearly similar as possible to the original disposition. The only condition we were unable to realize was the same degree of mechanic and magnetic stability. Nevertheless, we were able to find again the qualitative effects of magnetic induction already observed by Pender. We verified with a sufficient degree of approximation the proportionality of these effects with the speed of rotation of the discs and with the surface density of the charge carried around. We also found that the effects of the two discs were superimposed or annulled each the other, according to their relative directions of rotation, and that the effect obeyed the law of the distance between the discs and the induced coil. As to the order of magnitude of the quantities observed, the deflections obtained varied from 58 to 800 mm., according to the values of the other quantities involved, on a scale placed 4 meters from the galvanometer.

Repetition of Cremieu's Experiment. — Cremieu in the meantime mounted his induction experiment, the conditions being almost identical with those of the original experiment. However, the galvanometer previously employed had been destroyed, so we were obliged to use another. Also the conditions of stability were not so good as in the original installation.

Although the effects due to charging and discharging the disc at rest were not entirely eliminated, we obtained with a sufficient degree of certainty the same negative results as in the original experiment.

Verifications in Cremieu's Experiment. — In these two experiments we employed continuous metal discs turning between continuous parallel condensing plates. In the course of the repetition of Pender's experiment we observed the following fact. When a voltage was employed sufficiently high to cause sparks to spring across from the discs to the condensing plates, the magnetic effect observed

[1] Phil. Mag., 6th series, Vol. 2, p. 169, 1901 et loc. cit., PHYS. REV., Vol. XV., p. 291, 1902, et loc. cit.

fell practically to zero, whereas the potential of the discs, as measured by an electrometer connected thereto, showed a decrease of less than a tenth of the value observed just before the sparks began. In Cremieu's apparatus it was impossible to see such sparks if they should occur, as the disc turned in a completely closed box of cast iron. However, in spite of the fact that the disc and condensing plates were covered with a thin layer of caoutchouc, it was possible that such sparks might occur, as the distance between these pieces was very small.

Only a galvanometric measurement of the quantity of electricity taken by the disc at each charge or the quantity given up at each discharge could decide this point. We made this measurement, employing voltages from 1,000 to 5,500 volts. For 1,000 volts it is certain that no spark can occur across 3.5 mm. of air. As the voltage is increased, one should obtain currents proportional to the voltage as long as no sparks occur. If such sparks take place, the charging current should increase more rapidly, the discharge current less rapidly, than the voltage. Measurement showed that up to 5,500 volts, the maximum potential employed, there was a strict proportionality between the charge and voltage.

The cause of Cremieu's negative results was not then in this.

Verifications in Pender's Experiments. — In the series of experiments on "open currents" made by Cremieu[1] in 1902 with the aid of M. J. Jarval, certain peculiar magnetic effects were observed in the neighborhood of a node of electrical oscillations such as is formed by the turning disc in convection experiments. The conditions for the production of these effects are sufficiently similar to the conditions in Pender's experiment, that one might believe that the effects observed were due to such oscillations and not to convection itself.

This hypothesis was rendered still more plausible by certain effects observed when the discs are charged or discharged at rest. These effects, though extremely irregular, are capable of giving to the galvanometer connected with the induced coil deflections of more than 100 mm. Moreover, these effects are considerably augmented by the smallest hole in the electric screen of tin foil which

[1] C. R., Vol. CXXXV., p. 27, 1902; and Jour. de Phys., Dec., 1902.

protects the coil. Again, as the tin foil is wrapped around the winding of the coil only, thus leaving the center of the coil open, any electric oscillation through the coil will suffer little damping.

To ascertain if such oscillations played any rôle, we placed Pender's coil in a box of brass with walls 2 mm. thick, entirely closed. This diminished considerably the perturbations caused by charging and discharging the discs at rest, but the magnetic effect due to the movement remained practically of the same order to within 10 per cent., that is, as close as we could observe under the unstable conditions of the experiment.

However, in Cremieu's negative experiments, the iron box in which the disc turns can arrest *every* magnetic oscillation tending to traverse the coil ; this box constitutes a perfect magnetic screen, which is not the case with the brass box in which we enclosed Pender's coil. In fact, it is well known that damped electric waves, such as those which occur around a body whose charge is rapidly varying, traverse without considerable alteration even very thick conducting screens. To verify this point, it would therefore have been necessary to put a magnetic screen around Pender's coil.

The following method of procedure is, however, much simpler. If the effects observed by Pender are due to an oscillatory phenomenon, they must certainly be modified by any change in the circuits serving to charge and discharge the discs.

We placed in these circuits liquid resistances. They did not cause the effect to disappear ; they merely diminished considerably the perturbations at rest.

Again, we placed in parallel with the discs a variable capacity. The capacity of the discs was 200 C.G.S. electrostatic units. The capacity in parallel was an air condenser and could be varied by sixths from 166 to 1,000 C.G.S. This time the effect in motion was considerably diminished. The diminution was approximately proportional to the capacity in parallel with the discs. At the same time we noticed a diminution in the capacity assumed by the discs. (In all our experiments we used as the source of charge M. Bouty's high potential storage battery, capable of giving as much as 14,000 volts.) The potential was measured by an electrometer connected automatically to the discs at the moment they were

charged. However, there was not a proportionality between the diminution of the voltage and the diminution of the deflection. The deflection dropped to one third its original value, whereas the decrease in voltage was only 10 per cent. However, we discovered that the discs no longer became completely discharged, and that the charge remaining on the discs was proportional to the capacity in parallel.

These facts led us to study the duration and number of contacts necessary to charge or discharge a condenser completely. A cylindrical rod R, Fig. 1, of ebonite, 120 cm. long, was placed vertically in such a manner that a brass ring B, 1 cm. high, could fall freely along R. At 10 cm. above the lower end of the rod were placed two metallic brushes C and D, opposite each other. The brass ring B in falling made contact between C and D for any desired interval of time, depending upon the length of the drop. The condenser employed was the same as that put in parallel with the discs. The source of electricity was the high potential storage battery of M. Bouty, the capacity of which can be considered infinite in comparison with that of the condenser.

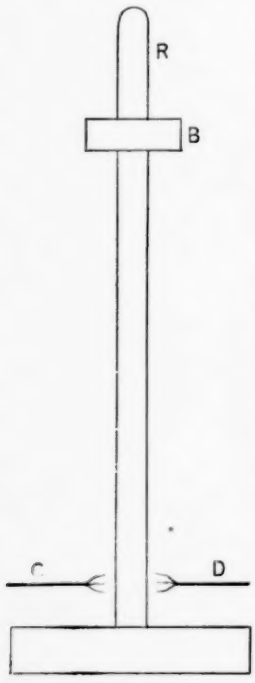

Fig. 1.

The experiment was made as follows: First, was measured the potential of one pole of the battery when the other was earthed. One plate of the condenser was connected to earth, the other to the brush D. The brush C could be connected either to the free pole of the battery or to the earth. First, let C be connected to the battery, and let the ring fall, contact will be established between C and D as the ring passes. The duration of the contact could be varied between $\frac{1}{10}$ and $\frac{1}{90}$ of a second. After the fall the potential communicated to the condenser was measured by an electrometer connected permanently thereto. Under these conditions we observed the following:

1. The potential assumed by the condenser was always inferior to that of the battery. This difference was inversely proportional to the time of contact and directly proportional to the capacity of the condenser.

Next the condenser was charged by a prolonged contact with the battery and then insulated from it, C was connected to the earth, and the ring B dropped. The electrometer connected to the condenser then indicated if the condenser retained a charge after contact. We observed the following :

2. The condenser always retained a residual charge, the value of which was directly proportional to the capacity of the condenser and inversely proportional to the duration of the contact.

However, the difference between the potential of the battery and that taken by the condenser after contact was much greater, all else being the same, than the residual potential after the discharge.

These facts explain satisfactorily the diminution of the effects of induction of the discs in motion when the capacity is put in parallel with the discs. The presence of this capacity decreases the variation of the charge at each reversal of the commutator, and, in consequence, the effects of induction, which are proportional to this variation ; further, the indications of the electrometer, which gives the maximum potential assumed by the discs, cannot be affected to the same degree.

Consequently, it appears that the effects observed in Pender's experiment are due to the movement of the charged discs, as demanded by the theory of electric convection.

Verifications Without Fixed Condensing Plates. — To eliminate entirely the questionable rôle of the fixed condensing plates, we made the following modification.

Pender's two discs, diameter 31 cm., were placed opposite each other at 1 cm. apart, and arranged to turn in opposite directions. Concentric with the two discs was placed a coil 34 cm. in diameter and 3 cm. thick. This coil, which was entirely enclosed in a brass sheath, consisted of 1,300 turns of copper wire with a total resistance of 60 ohms. The commutator was arranged so as to connect this coil to the galvanometer synchronously with the charging and discharging of the rotating discs. The deflections of the galva-

nometer under these conditions was much less than in the former experiments, on account of the considerable decrease in the capacity of the two discs thus arranged. In spite of that, however, the deflection was sufficiently large to be measured, and the agreement between the observed and calculated deflection was quite satisfactory.

Here we stopped our experiments with the induction method. These experiments can, in fact, give no information in regard to the nature of the magnetic field produced by the movement of the discs, whereas it is essential to know if this field is permanent with the rotation, or is merely an instantaneous field produced at the moment of charging or reversing the charge, resulting from magnetic perturbations thereby set up.

Experiments on the Direct Magnetic Effect. — First we utilized the same arrangement as in the preceding induction experiment. The coil was replaced by a delicate astatic system enclosed in a metallic tube so arranged that the lower needle of the system was 1 cm. above the upper edge of the discs, in a plane equally distant from the two. The tube was protected from the air currents caused by the rotating discs by a large sheet of mica properly placed. The sensibility of the system was determined by a test coil, of a radius equal to the mean effective radius of the disc. The reversal of a current of 10^{-4} ampères in this coil gave a deflection of 20 mm. on a scale 4 meters distant. The convection current possible to realize in this disposition was only 2.5×10^{-5} ampères, and in consequence the deflection expected was scarcely 5 mm. Hence the experiment was purely quantitative. However, there was no doubt as to the effect. At each reversal of the sign of charge on the turning discs, the system was deflected in the expected direction, but it was impossible to say, on account of the unsteadiness of the needle, whether the deflection was permanent or only an impulse.

It is to be noted that in spite of the smallness of this result, it is nevertheless of great importance. Any perturbing effects due to the fixed condensing plates or to currents which might circulate in the moving discs, are entirely eliminated, because the first are suppressed, and the second would destroy each other, as the discs turn in opposite directions.

In the above experiment, only a very small part of the discs is effective. The needle is in a very unfavorable position, for the magnetic field over the edge of the discs is very unstable, and further the needle is very poorly protected against the air currents, which cause considerable unsteadiness. These considerations led us to return to Rowland's original method.

A single disc turning in a horizontal plane was enclosed in a box of ebonite, the inside of which, covered with tin foil, formed the condensing plates, thus increasing considerably the capacity of the disc. This arrangement allowed us to put the astatic system very close to the disc, in a region where the magnetic field is at the same time horizontal and quite constant; and finally, the effects of the air currents were entirely eliminated. We were therefore enabled to give the needle a much longer period, without having to fear so much the mechanical perturbations which might be produced during the time necessary for the system to complete its swing.

This time the deflections obtained were considerable, 20 to 30 mm., of the order and in the direction expected. But it was still impossible to decide with certainty whether the deflections were permanent or simply impulses.

Pender observed in his former experiments,[1] in which a perfect stability was realized, deflections which were undoubtedly permanent, but he did not know at that time of the magnetic effects which are produced in the neighborhood of nodes of electric oscillations such as formed by the discs, a description of which Cremieu has recently given.[2] He was therefore not absolutely certain that the deflections he observed were not due to such oscillations, the effects of which might be easily confused with those due to a permanent magnetic field presumably due to the convection current.

These considerations led us to modify both our mode of operation and the system used to detect the magnetic field.

New Mode of Operation. — In all the experiments performed up to the present by other experimenters and ourselves, the effect observed was always that resulting from charging or reversing the sign of the charge on the discs already in rotation. Of course, care

[1] PHYS. REV., Vol. XV., Nov., 1902; Phil. Mag., Vol. V., Jan., 1903, p. 34.
[2] C. R., Vol. CXXXV., p. 153, 1902.

was always taken that there was never an effect produced by charging or reserving the charge on the discs at rest, which condition can be realized by properly enclosing the astatic system (or galvanometer circuit) in a suitable conducting screen. However, the action of such a screen is both electrostatic and electromagnetic. Besides, it is well known that in the neighborhood of a node of electric oscillations there are produced rapidly damped magnetic waves capable of demagnetizing a magnet; and such a demagnetization can result in a permanent change of the position of equilibrium of an astatic system. Moreover, it is unknown what effect the movement of the body forming the node for these oscillations can have upon the oscillations themselves. Consequently one cannot assume *a priori* that a screen which is sufficient to suppress any effect of these oscillations when the discs are at rest will also be sufficient when the discs are in motion.

The following mode of operation appears to us to avoid all these inconveniences. First, charge the disc at rest, care being taken that this operation produces no effect on the magnetic system. Then insulate the disc from the charging source and set it in motion. The magnetic system should then, if the effect of convection exists, take a deflection increasing with the velocity, permanent for any given velocity, and returning to zero the moment the disc is stopped. An electrometric measurement of the potential of the disc before and after the movement will show if there has been any sensible leakage.

As ordinary astatic systems are poorly suited for such an experiment, we attempt to construct a system of a different kind.

Experiments with New Magnetic Systems. — It is difficult to realize, in most laboratories, a magnetic stability sufficient to allow of the measurement of magnetic fields as low as 10^{-6} C.G.S. electromagnetic units, the order of the field in most convection experiments. Further, from the very nature of its construction, a sensitive astatic system always tends to drift in a certain definite direction, due to the slow and unequal demagnetization of the needles which compose it. Another cause of instability is that the direction assumed by such a system is determined by the difference between the earths' field and that of the compensating magnets; consequently any slight

variation in either of these fields will produce a considerable change in the position of equilibrium of the system. In particular, if the field to be studied is produced by oscillatory phenomena more or less damped, astatic systems become practically useless, since the demagnetizations which accompany these damped oscillations not only affect the system itself but also the directing magnets. Finally, in the case where the field to be studied is very feeble, it seems advantageous to increase the effect by using strong magnetic poles, but it is well known that the sensibility of an astatic system is independent of the moments of the magnets which form it.

All these inconveniences can be avoided, without decreasing the sensibility, by employing an extremely simple system. This system consists essentially of a light horizontal beam carrying at one end a vertical magnet, and at the other a non-magnetic counter-weight of brass. The whole is supported by a long, fine metallic wire fixed to the center of the beam. If the magnetic axis of the magnet is exactly vertical, the couple to which this magnet is submitted due to the earth's field will produce no action on the torsion wire. The beam will therefore take up a position due only to the torsion of the wire which supports it. To adjust the system to this condition, the magnet is first replaced by a non-magnetic needle of the same form and weight, and the period of oscillation of the system is determined. The magnet is then replaced. In general the period of oscillation becomes much shorter and the beam takes up a new position of equilibrium. However, by adjusting the counterweight, an exact verticality of the magnetic axis of the magnet can soon be attained, which is indicated by the system again taking the same period of oscillation as when the magnet was replaced by the non-magnetic needle. A magnetic system is then realized which is directed only by the torsion of the supporting wire.

With systems of this kind we have obtained very great sensibilities. For example, for a magnetic pole of 20 C.G.S., supported by a silver wire 95 cm. long and .025 mm. in diameter, we obtained a deflection of 34 mm. on a scale 2 meters distant, for a variation of field of 10^{-6} C.G.S.; the period of oscillation being about 60 seconds. When this system was placed near the disc in a suitable screen, it was absolutely unaffected by the starting of the motor and the discs.

However, an unexpected effect prevented us from obtaining what we had hoped. At the end of about two minutes of rotation of the disc, over which the system was placed, the system began slowly to deflect, the deflection increasing as time went on, thus rendering all observations impossible. At first we attributed these deflections to air currents set up in the screen containing the system by the heating of the condensing plate over the disc. This plate became quite warm on account of the violent vibrations caused by the rapid motion of the disc. But the "dummy" system, with the non-magnetic needle gave no such deflections; hence the effect could not be attributed to air currents. We finally concluded that the cause of this effect was the following: The rotation of the disc immediately below the pole of the magnet causes Foucault currents to be set up in the gilded surface, and the reaction of these on the magnet produces either a slow demagnetization or a change in the position of the magnetic axis of the needle with respect to its geometric axis. But be the cause what it may, we were forced to abandon the use of such systems.

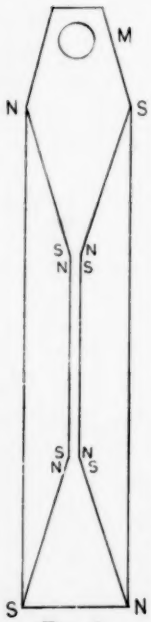

Fig. 2.

Returning then to the ordinary systems, we constructed a very simple one in the following manner. Three pair of very fine steel needles magnetized to saturation were arranged upon a thin sheet of mica as indicated in Fig. 2. This system was 6 cm. long and weighed about 500 mg. It was suspended inside a metallic tube from a silk fiber about a meter long. A test current sheet, constructed in such a manner as to give a distribution of current similar to that which is produced by the turning disc, was used to determine the sensibility of the system. When this test sheet was put in place of the disc and a current of 10^{-4} ampères reversed through it, the system was deflected 120 mm. This sensibility was amply sufficient, since the convection current realized in the experiments was between 3 and 5×10^{-5} ampères.

But even with this system we were not able to realize exactly the experiment which we have described. The discs of micanite

which we employed are magnetic, and very unequally so at different points. Consequently the system would take different positions of equilibrium depending upon the part of the disc immediately under it. However, if the disc is given a velocity such that it makes at least one complete turn during the time required for a complete oscillation of the system, the latter will take a mean position of equilibrium, which will not change when the velocity is augmented.

We therefore proceeded as follows: The disc was given a slow rotation, the equilibrium position of the system noted, the disc then charged, and the position of equilibrium again noted. Thanks to the liquid resistance in the charging circuit, the change in the equilibrium position was too small, if any, to notice. Then the speed of rotation was increased to its maximum, maintained at such for some time, and the position of equilibrium again noted; finally, the velocity was reduced to its first value and another reading taken.

In this way we assured ourselves of the following.

There was a deflection of the system when the velocity was increased, in the direction demanded by the theory of electric convection.

The deflection was permanent.

It accorded quantitatively to 10 or 20 per cent. with the calculated deflection.

From all the foregoing results, corroborated by those previously obtained by Pender, we can conclude that,

A charged disc, having a continuous metallic surface, turning in its own plane between two fixed condensing plates, parallel to this plane, produces a magnetic field in the direction and of the order required by the theory of electric convection.

There now remained two questions for us to solve.

1. Are the magnetic effects thus obtained due to an actual *entrainement* of the charge by the moving metallic surfaces, or can they be attributed in any way to conduction currents, open or closed, produced by the relative movement of the disc and the condensing plates?

2. What cause concealed from Cremieu the effects observed by Pender in his previous experiments and also observed by us in common in the experiments just described?

Experiments on the Entrainement of the Charge by Continuous Metallic Surfaces. — To answer the first question we first undertook an experiment the idea of which is due to Hertz and the first attempt at realization to Rowland. Consider a plane ring, continuous and conducting, represented schematically (Fig. 3) by the circle *NRMP*.

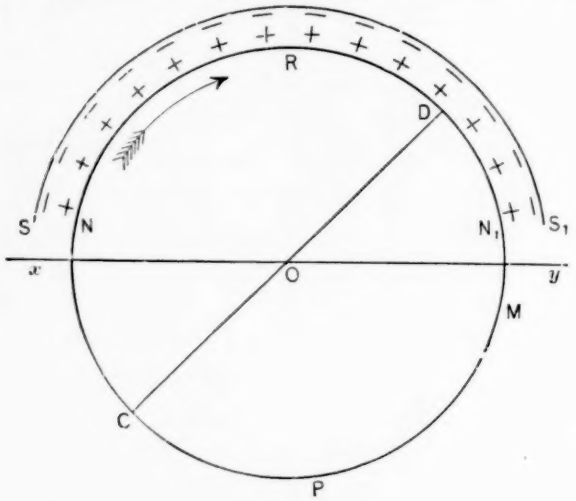

Fig. 3.

This ring can turn in the direction of the arrow under a fixed condensing plate represented by the arc SS_1, covering only a portion of the moving ring. Let the arc covered by this condensing plate be $1/n$th of the whole circumference, let ρ be the distance between the planes of the two plates *NRMP* and SS_1, γ the linear velocity of any point on the ring *NRMP*. Suppose SS_1 charged to a negative potential K, and *NRMP* connected permanently to the earth. As each element *ds* of *NR MP* arrives opposite S it will become charged by influence. If the hypothesis ordinarily assumed is true, this element should carry its charge along with it. In other words, this charge will be displaced with reference to the electrostatic field between *NRMP* and SS_1, which remains fixed in space. When *ds* arrives opposite S_1, the charge on it is no longer retained by influence. As this is true for all elements of *NRMP* there will result from the movement a constant difference of potential between the two points N and N_1 on the moving ring opposite S and S_1; consequently conduction currents will be set up in the ring, and

will be distributed according to the relative resistances of the sectors NRN_1 and NPN_1.

A magnet placed above R will therefore be subjected to magnetic field due to two causes, first, that due to a convection current in the direction of the arrow, second, that due to a conduction current in the opposite direction. If the needle is placed over P it will be acted upon by the conduction current only. Moreover, the sum of the conduction currents in NRN_1 and NPN_1 should be equal to the intensity i of the convection current.

Let σ be the surface density of the charge on $NRMP_1$ then the intensity of the convection current due to a ring 1 cm. wide is

$$i = \sigma\gamma.$$

But

$$\sigma = \frac{K}{4\pi\rho},$$

Hence

$$i = \frac{K\gamma}{4\pi\rho}.$$

Let R be the total ohmic resistance of the conducting ring. Then the resistance of NRN_1 is R/n and that of NPN_1 is $R(n-1)/n$. The conduction current in N_1RN is then evidently

$$-i_1 = -i\left(\frac{n-1}{n}\right)$$

The magnetic field at a distance above R great with respect to ρ will be due to the difference between i and i_1, that is to

$$i\left(1 - \frac{n-1}{n}\right) = +\frac{i}{n}$$

Similar reasoning shows that the field over P is proportional to $-i/n$.

Hence a magnetic system placed over P or over R will show magnetic effects equal and in opposite directions.

Rowland tried this experiment, but the smallness of the effect expected prevented his obtaining satisfactory results. We tried the experiment giving $1/n$ the value $\frac{1}{2}$. The conducting ring was

thin gold leaf on an ebonite disc, the width of the ring being 5 cm. and the mean radius 14.5 cm. The ring was divided by concentric scratches into strips 1 cm. wide. The calculated intensity of the convection current was 4×10^{-5} ampères. The deflections of an astatic system placed over P were in the direction expected and accorded quantitatively with the calculated values within the usual approximation in our experiments, *i. e.*, 10 to 15 per cent.

However, by slightly modifying the experiment, we were able to measure the quantity of charge carried around with great precision. If the two points on the disc immediately under S and S_1 are connected to a galvanometer the resistance of which is of the same order as the halfs of the ring $NRMP$, there will flow through the galvanometer a current easily measureable, since it will be a considerable fraction of the convection current, that is to say, in this experiment, of the order of 0^{-5} ampères. The gilding on the disc can be made extremely thin, having a resistance of from 4 to 6 ohms per square centimeter of surface. The resistance of one half the ring on our disc was 6 ohms. We employed a galvanometer of the D'Arsonval type (Hartman and Braun) having a resistance of 4.38 ohms and sensitive to 10^{-7} ampères. Hence for a convection current of 4×10^{-5} ampères, the current flowing through the galvanometer will be of the order of 1.6×10^{-5} ampères, and the resultant deflection about 160 mm., a deflection susceptible of great accuracy of measurement.

The first experiment was to place two fixed metallic brushes so as to rub on the ring under the points S and S_1. However, the friction of the brushes against the uncharged disc was sufficient to produce a deflection of about 100 mm. Moreover, the gold leaf was rapidly rubbed away, and the resistance therefore rapidly increased, so that all we could obtain from this method of procedure were rough qualitative results.

But by making yet another modification we were enabled to make quite accurate measurements. This was accomplished by employing the following exceedingly simple and effective contacts. At the extremities AA of the moving axle (Fig. 4), coinciding exactly with the axis of rotation and insulated from the axil, were fixed two very fine copper wires. Two small glass tubes, TT_1, all but sealed up at

T_1 so that the opening there was just a little larger than the diameter of the wire *aa*, were so placed that the wires turned in the openings without touching the walls of the tube. Before these tubes were placed in position they were filled with mercury and electrodes sealed in the outer ends. These contacts proved extremely satis-

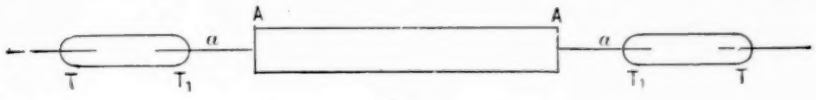

Fig. 4.

factory. Their resistance when the axil was in motion remained practically the same as when the axil was at rest, an hour's running producing an increase of resistance of less than five hundredths of an ohm.

To return to the convection experiment. Two points on the ring *C* and *D*, diametrically opposite, were connected permanently to the two wires *aa*, which in turn were connected through the contacts to the galvanometer. If now SS_1 is charged and the disc set in motion, there will be produced in the ring conduction currents distributed as explained above. But as *C* and *D* move with the disc, between these two points the difference of potential will be alternating, and consequently in the galvanometer there will be an alternating current, and hence no deflection. However, if in the galvanometer circuit is placed an interrupter actuated by an eccentric on the axle of the disc, and arranged so as to close the galvanometer circuit during the half period while *CD* turns from *PR* to *RP*, a unidirected current can be obtained, for the difference of potential between *C* and *D* during this interval passes from 0 to 0 through a maximum corresponding to the moment when *CD* is parallel to *xy*. The galvanometer should then show a deflection. If the eccentric is shifted 90° so as to close the galvanometer circuit a quarter of a period later, this deflection should become nil, because during the time the circuit is closed the difference of potential passes from a positive to an equal negative maximum. Between these two positions of the eccentric all intermediate deflections should be obtainable.

This experiment proved unusually satisfactory. The maximum deflection was quite large and the measurements made agreed within 20 per cent. with the calculated values, which, considering

the numerous approximations which must necessarily be made in determining the various constants (surface density of the charge, relative resistances of the various parts of the circuit, speed of rotation) is all that could be desired.

From these results we may therefore conclude that *a charged metallic surface moving in its own plane in the presence of fixed parallel metallic surfaces carries its charge along with itself.*

Experiments with Sectored Discs. Cremieu's "Open-current" Method.— Having thus solved in the affirmative the question of the existence of a magnetic effect produced by the rotation of a charged continuous disc, and verified directly the *entrainement* of the charge, we returned to the experiments made with sectored discs.

During the past year, Cremieu[1] obtained with discs formed of insulated sectors magnetic effects which were quite irregular and without any quantitative relation to the intensity of the convection

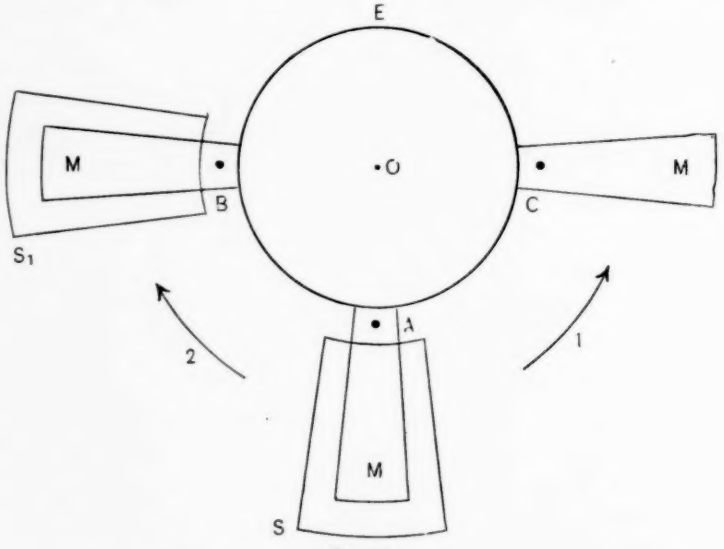

Fig. 5.

current, which, in these experiments, was measured directly. The apparatus consisted of a core of ebonite 24 cm. in diameter, carrying 18 sectors of micanite, 13 cm. long, separated from one another by 2 cm. of air. Only the outer portion of the sectors was

[1] C. R., Vol. CXXXVI., p. 27, 1902.

gilded, the gilt covering a width of 5 cm. The moving sectors M (Fig. 5) passed between two charged fixed sectors S, and at the same time touched a brush A connected to the earth; they therefore became charged by influence. They then passed from under the brush A and the fixed sectors S and came around under the astatic system E. Beyond the system they met a second brush B connected to the earth, thus becoming discharged. By placing a galvanometer between A or B and the earth, one could measure the charging current or the current resulting from the discharge.

Certain peculiar phenomena of solid dielectrics, first observed by Cremieu, and to which we shall return farther on, led us to employ in our joint experiments micanite sectors entirely gilded, so as to avoid all penetration of charge into the naked micanite. The astatic system was suspended in a metallic tube connected to earth. To protect this tube from the electrostatic action of the moving sectors, which were at a very high potential, Cremieu employed a paraffined tube of mica fixed directly on the metallic tube. To this disposition there are two objections. Since the bottom of the tube is small relative to the area of the moving sector, at the moment when the sector comes under the tube there is a considerable increase of the capacity of that portion of the sector directly under the tube. Consequently to this point there will be a flow of charge by conduction currents distributed in a manner impossible to calculate, which can act upon the astatic system. Secondly, in consequence of the peculiar phenomena of the dielectric to which reference has been made, the mica protector of the tube is submitted to a penetration of charge from which may result considerable perturbations. To avoid these inconveniences, in some measure at least, we left the metallic tube bare, but interposed between it and the moving sectors a large sheet of paraffined ebonite, which touched neither the tube nor the sectors.

In these conditions, the passage of the charged sectors under the astatic system produced deflections qualitatively in accord with the effects expected from the theory of convection, and properly varying with the density of the charge and the velocity. As to quantitative agreement, that was not very satisfactory, but in this form of experiment the distribution of the charge on the sectors is

too indefinite to allow of even a rough approximation. Again, as the sectors rotate in the open air, it is impossible to eliminate completely the effects of the air currents, which render the system too unsteady for accurate readings to be possible.

There now remained to find the cause of the repeatedly negative results of Cremieu's experiments.

Essential Difference Between the Negative and Positive Experiments. — An analysis of the details of the negative experiments led us to see that they all differed from the positive experiments in this, that in the negative experiments the moving charged surfaces and the condensing plates, when there were any, were always covered with a thin layer of some dielectric, usually caoutchouc. In his experiments Cremieu found it desirable to realize as high a surface density of the charge as possible. To do this, he placed the turning disc and fixed condensing plates very near together, and the caoutchouc layers were for the purpose of preventing sparks between plates and discs. *A priori*, one could see no inconvenience resulting from the presence of the caoutchouc; in fact, one would naturally suppose that its presence would prevent leakage and ensure a more perfect *entrainement* of the charge, for, as the charge is carried on the surface of the dialectric in contact with the conductor, any slipping which might occur would be rendered less probable.

To test the truth of these considerations we tried the effect of covering the moving sectors in the above experiment with a layer of caoutchouc. There resulted a considerable diminution of the magnetic effect, without, however, the intensity of the convection current, as measured by the quantity of electricity going on or leaving the sectors, showing a corresponding diminution. Moreover, the nature of the deflections of the astatic system changed. At first quite distinct for the two signs of the charge, they rapidly became smaller, and at the end of several reversals became scarcely perceptible, especially when the sectors were charged positively. For the negative charge the diminution was less. Further, the deflections ceased to be proportional to the potential of the fixed sectors for voltages above 2,000 volts. Also, the charge going on the sectors became less than it should be according to calculation, and above a certain definite voltage, the charging current remained

practically constant, no matter how much the voltage was increased.

These facts made clear the cause of Cremieu's negative results. It is to be noted, that in 1900, in his first experiments on open currents, Cremieu observed a part of these phenomena. However, as he was eager to complete his experiments on convection, he did not stop to investigate fully these effects.

We now undertook a systematic study of the rôle of the dielectric, but due to the brevity of the time at our disposal, we have not been able to carry this study very far.

As of the first importance in regard to the theory of convection, we verified the following points:

1. When the continuous discs were covered with caoutchouc, the magnetic effect diminished and presented the dissymmetry of sign observed with the sectored disc. In these experiments the charge on the discs can be determined only by measuring the potential. These measurements showed that there was the same lack of proportionality between the potential and the magnetic effect observed.

2. A thin sheet of paraffined mica fixed on the surfaces of the discs produced the same effects as the caoutchouc.

3. In Pender's induction experiment, we found that the mica diminished considerably the effects observed. For example, in one series of measurements we obtained the following deflections.

	Mm.
Discs bare, condensing plates bare,	140
Discs covered with mica, condensing plates bare,	100
Discs and condensing plates covered with mica,	15

These facts show clearly the experimental cause of Cremieu's negative results, but they do not *explain* them. Indeed, the rôle of the dielectric seems difficult to unravel from any *a priori* considerations.

From the facts observed in 1900 and in our joint experiments it seems legitimate to conclude that, when solid dielectrics are submitted to considerable penetrations of charge, they act as if they suppressed the electrostatic influence between the conductors which they separate. Moreover, for each kind of dielectric the penetration of the charge and the suppression of electrostatic influence which

results therefrom occurs at a certain well-defined voltage of the charged conductor — being independent of the value of the electrostatic field produced by this voltage. For mica the critical voltage is in the neighborhood of 4,300 volts, for sulphur between 8,000 and 12,000 volts; for ebonite, glass and paraffine the critical voltage is above 12,000 volts.

In the negative convection experiments the action of the charge on the metallic surfaces was rendered nil by the screening effect of the dielectric, or possibly by the neutralizing effect of the very considerable charges absorbed by the dielectric. But these are only suppositions — only a detailed, systematic study of these dielectric effects can clear up this very delicate point.

Study of the Amount of Charge Carried Around Under Various Conditions in the "Open-current" Experiments. — To complete our researches we undertook a study of the apparent amount of charge carried around in the open-current experiments, with the object first, to make sure if the third fundamental condition laid down at the beginning of this article was realized, and secondly, to clear up a little more the rôle of the dielectric.

The apparatus employed (Fig. 5) was the disc with the ebonite core *OE* carrying 18 peripheral sectors *M*. These passed between two fixed sectors *S* and at the same time touched the brush *A*; then, according to the direction of rotation, they either became directly discharged by the brush *C*, or first passed between two supplementary fixed sectors S_1 and then were discharged at *C*. The sectors S_1 formed with each sector *M* a capacity equal to that formed by the sectors *S* with *M*. These sectors S_1, connected to an electrometer, permitted us to see what takes place, of an electrostatic nature, in the air around the moving charged sectors. An experiment consisted in measuring galvanometrically the quantity of charge taken up by the sectors as they came under *A*, and the quantity of charge given up at *C*; also the reading of the electrometer connected to S_1 was noted. The results were as follows:

1. *Moving Sectors M Bare, Fixed Sectors S Bare.* — (*a*) Whatever was the sign of the charge, the *discharge current* was almost invariably *superior* to the *charging current*, the excess being variable, but in some cases was as high as 15 per cent. This dissymmetry was the greater when the moving sectors were positive.

(*b*) On the sectors S_1 the following peculiarities were observed: Suppose the disc in rotation, in the direction of the arrow 2, the sectors S_1 connected to an electrometer, and all the rest of the apparatus to earth. No matter what was the velocity of rotation no deflection of the electrometer was observed. However, when the sectors S were charged, so that the moving sectors M were charged as they passed between the sectors S, the electrometer immediately took a sudden deflection, which gradually increased, the more rapidly the faster the rotation. The sign of the charge on the electrometer was the same as that of the charge on the moving sectors, while the charge on the sectors S_1 was of the opposite sign, as should be expected. However, if the sectors S and M were then earthed, the deflection of the electrometer did not fall to zero, but indicated a residual charge on the sectors S_1, which charge was of the same sign as that of the moving sectors.

These facts seem to show that the rotation of the charged sectors produce in the surrounding air electricity of the same sign as that they carry. In accord with this assumption is the fact that the charging current is inferior to the discharging current. In this connection attention may also be called to the fact that the loss of charge of a body in air is considerably less when the the body is in rapid movement than when the body is at rest. This phenomenon was first observed by Matteucci.[1]

At present we merely state these facts with no attempt at an interpretation. It is impossible to say whether this production of charge should be attributed to the movement itself or rather to sudden variations in the field caused by the violent shocks given to the air by the rapid movement of the sectors. It should be added that we tried to detect a possible production of charge by placing in the air between a charged disc and earthed condensing plates small metallic brushes connected to an electrometer. The electrometer showed no deflection when the disc was set in rotation.

2. *Moving Sectors M Bare, Fixed Sectors S Covered with Mica.*— We found the same dissymmetry between the charge and discharge current as in the preceding case, and in the same direction. Further, for voltages above 3,000 volts on the fixed sectors S_1 the charging

[1] Ann. de Chimie et de Phys., 3rd Series, Vol. XXVIII, p. 385.

current did not increase proportionally with the voltage on the fixed sectors. When this voltage reached about 4,500 volts, the charging current appeared to attain a maximum, which could not be exceeded even for a very great increase in the voltage.

Also, when the inducing sectors S were earthed after having been submitted to a charge for several seconds, the moving sectors continued to take up a charge as they passed under S, as indicated by a current continuing to flow through the galvanometer, but of the opposite sign to that impressed upon S, and this after-effect continued even for several hours thereafter without noticeable diminution. These phenomena were extremely irregular, the size of the supplementary currents seeming to depend upon the previous state of the mica and the duration of the experiments.

At the sectors S_1 the same phenomena were observed as in the preceding case.

3. *Moving Sectors M and Fixed Sectors S Covered with Mica.* — As long as the potential of the fixed sectors S was below 2,000 volts no anomalies were observed. Above 2,000 volts the charge current became considerably greater than the discharge current. The dissymmetry attained even as much as 30 per cent. In the neighborhood of 4,500 volts, the proportionality between voltage and charge current ceased, as in the preceding case. There were also supplementary currents after the inducing sectors were earthed. These could attain as much as 50 per cent. of the value of the normal currents when the inducing sectors were charged. But there was this peculiarity, that for the moving sectors charged positively they were in the opposite direction to the normal currents, while for the moving sectors charged negatively they were in the same direction as the normal currents. The sectors S_1 took up *no charge by influence*. Even more so than in the former case did all these irregularities depend on the previous state of the dielectric and the duration of the experiments.

All this shows how complex the phenomena are, and that only a prolonged study can fully explain them. This much seems certain, that there is a penetration of charge in another sense from that usually understood by the expression. That which seems to prove this most conclusively is that, in the third case above, the supple-

mentary currents are obtained only when the two brushes A and C both make contact with the sectors. If either of the brushes is suppressed, there is no longer a supplementary current. If the supplementary currents were due merely to the ordinarily considered absorbed charge, this charge would gradually flow out through the brush left in contact, and therefore produce a current in the galvanometer, without the presence of the second brush being necessary. It seems to us that the mechanism of the supplementary currents is analogous to that of an electrophorous, the mica on our sectors in some way or other, being electrolyzed.

It is essential, in any case from the point of view of the accepted theories, to know why the dielectric diminishes or suppresses the magnetic effect, and what is the nature of the action of penetration.

Conclusions. — All that can be said at present of these accessory phenomena is that they do not permit us to affirm with certainty that the third fundamental condition of a correct convection experiment is exactly fulfilled, particularly with sectored discs. But the following conclusions are certainly legitimate.

1. A charged continuous metallic surface turning in its own plane opposite fixed parallel condensing plates carries its charge with itself.

2. The *entrainement* of this charge produces a magnetic field in the direction demanded by the assumption of a magnetic effect due to electric convection, and in accord with the calculated value to 10 per cent.

3. Charged isolated sectors, moving in their own plane, without the presence of any condensing plates, produce a magnetic effect in the direction and of the proper size demanded by this same assumption.

It is not for us to say if these magnetic effects are really due to electric convection in the sense in which Faraday and Maxwell understood this expression, nor to decide if they are in accord with the fundamental hypotheses of the present theories.

PHYSICAL LABORATORY OF THE SORBONNE,
PARIS, April 15, 1903.

MEASUREMENT OF AIR VELOCITY AND PRESSURE.

BY A. F. ZAHM.

FOR general service in aërodynamic experimentation two kinds of anemoneter may be useful, one measuring air velocity, the other air pressure. Sometimes the velocity alone is required; again the pressure, either static or impactual, is mainly to be determined. In either case an accurate and faithful instrument is most desirable. Of course the impact can be calculated from the velocity, and *vice versa*, providing all the data are at hand for computing the density of the moving fluid. But when many exact determinations have to made, it is well to evade such calculations by choosing the appropriate anemometer; that is to say a velocity instrument for measuring velocity, and a pressure instrument for measuring pressure, or impact. One thereby saves much labor, and avoids the errors that may result from misreading the elements of density.

Suppose, for example, that one is finding the head resistance of various models in terms of the wind speed, observing the latter by means of a velocity anemometer. The wind speed may be kept constant throughout the research, but the resistance of any given model will vary day by day, and hour by hour. Every three degrees change of the thermometer, or eight millimeters change of the barometer, will alter the resistance one per cent. Even the variations of moisture may make a perceptible difference. So, in order to compare the resistances obtained at different periods, it is necessary to reduce them all to standard atmospheric conditions. This means a great deal of labor and many unwelcome errors. On the other hand, if a pressure anemometer is employed, all these troubles will be avoided. So long as its readings are kept constant the resistance of the models must remain constant, no matter how the atmospheric conditions may change. Thus all the observations of an extended research may be compared without reduction. In fact, for

impact measurements the pressure anemometer has three distinct advantages: (1) Computations are avoided, (2) readings of temperature, pressure and saturation are unnecessary, and errors from that source obviated, (3) the anemometer readings and model resistances may be plotted while the experiment is in progress, thus revealing observational errors which may at once be eliminated by taking new readings where required. In a similar way it may be seen that the velocity anemometer has equal advantages in many important investigations.

So the need of two kinds of anemometer is a very real one; and it would be well if they were standardized to measure accurately to say one per cent. throughout a wide range of velocities. At present no such instruments are offered in the market, though possibly some of the types in use could be standardized to that degree of accuracy, particularly the screw, the cup, and the pressure-tube varieties. But thus far the calibration of such instruments has proved difficult and not completely satisfactory. Indeed it would be an advantage if the anemometers were free from empirical constants, so as not to require calibration at all.

The present paper treats of the design and use of an anemometer[1] whose observed indications seem to conform to those computed for it from theory. The data submitted were obtained hurriedly, as

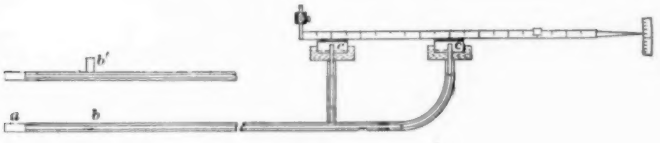

Fig. 1.

incidental to other measurements, but it is hoped they are accurate enough to establish confidence in this type of instrument; for it is easy to construct, convenient to use, adapted to a wide range of velocities, and has no empirical constant.

The anemometer is an adaptation of the Pitot tube, and consists essentially of a double pressure nozzle in connection with a delicate pressure gauge. The assembled parts are shown in Fig. 1.

[1] This instrument was exhibited before the Washington Philosophical Society, May 24, 1902.

A "pressure-tube" held along stream transmits the impact pressure of the air from nozzle *a*, the static pressure from nozzle *b*, to the cups *c*, of a differential pressure-gauge. These cups, symmetrical in size and placement, are inverted over coal oil, and counterpoised from a meterstick. A sliding weight, and a pointer measure the difference between the static and kinetic pressures at the double nozzle; and from this can be computed the speed of the air, if its factors of density are simultaneously observed; that is its temperature, pressure and saturation.

We may notice some elements of construction, then the theory and test of the instrument, and finally a few of its applications.

In designing the pressure-tube it is well to select the largest pipe that can be conveniently employed, so that the air within may quickly come to rest after any displacement. The one here shown has an inch and a quarter outer pipe, a three quarter inch inner pipe, each ten feet long and connected by three quarter inch rubber hose to cups measuring one hundred square centimeters in cross-section. The counterpoised beam responds quickly to the varying impulses of the wind, whereas with quarter-inch hose of ten feet length, the motion is tediously slow, owing to the great viscous resistance of the air in such narrow channels.

Almost any size and form of nozzle will convey the impact perfectly, providing it squarely face the wind. All sizes of pipe, from a lemonade straw to a milk-can show exactly the same pressure. All shapes, cylindrical, conical, hemispherical; all thicknesses of wall give identical results.

But the static nozzle must be designed with some care. It should occupy a part of the tube where the stream-lines are undisturbed, and be so shaped as not to deflect the air blowing over it when the cups are at rest. The nozzle here outlined has two longitudinal slots cut into opposite sides of the outer pipe, one foot from the end, each slot being one eighth inch wide by three inches long. As will be shown presently, this nozzle seems to give the true static pressure of the passing wind-current.

The cups may be rigidly attached to the beam, or hung like the pans of an ordinary balance. To avoid excessive stability, owing to the buoyancy of the cup walls, it is well to make them of the

thinnest metal practicable. For very delicate work it is essential that the liquid flow freely along the cup walls, so as to minimize the effect of surface tension; also the pointer excursion must be limited to a narrow range by suitable stops. If its swing is large the liquid trickling down the cup walls will materially alter the readings, unless a long time is allowed the pointer to come to rest.

The cups here shown have brass walls one hundredth of an inch thick, whose effective buoyancy is nearly one per cent. of the pressure on the internal cup base. The coal oil flows freely along their sides, practically eliminating the objection of surface tension. The pointer swings ten millimeters to either side of the zero, the cup displacement being two millimeters. With these limited excursions the trickling of the oil is practically unobjectionable, except for the most delicate measurements. For these, of course, the pointer can be still further limited. The buoyancy also, is unobjectionable. It is too small to impair the sensibility, and, since its effective lift per unit of surface is precisely equal to that of the air inside the cup, the total lift equals the product of the air pressure by the area of the external base of the cup.

The graduations are found to be convenient in use. The beam is a meter stick carrying a sliding weight and pointer, each of which gives the air pressure in milligrammes per square centimeter, that is to say in millionths of an atmosphere, approximately. The scale can be made to show less than one tenth of this when the pressure is sufficiently steady, which, however, does not occur in anemometry.

So much for the details of construction and practical operation.

The hydrodynamic principle of the double pressure nozzle is contained in Bernouilli's theorem. For present purposes, however, we may suppose the density of the air constant, its flow uniform and level. Bernouilli's theorem may then be stated as follows: At all parts of a stream-line the velocity head plus the pressure head equals a constant. Or otherwise:

$$\frac{\rho v^2}{2g} + p = p_0, \tag{1}$$

in which ρ, v, p are the density, velocity and pressure of the fluid at any point of the stream-line, and p_0 is the pressure due to the

total head, or pressure at a point where the velocity is zero. Hence, if the nozzle will measure the differential pressure $p_0 - p$, we can at once compute v therefrom.

To prove formula (1) let us take the general stream-line equation (Lamb's Hydrodynamics, §24) for adiabatic expansion:

$$\frac{v_1^2 - v_2^2}{2g} = \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2}\left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right],$$

v, p, ρ being the velocity, pressure, density, at any one point of the stream-line; v_2, p_2, ρ_2, like values at any other point. Let $v_2 = 0$ be the velocity at the impact mouth of the pressure-tube, v_1, the velocity in the unchecked stream, and write $p_1 = p_2(1 - x)$. Then the general equation can be written:

$$\frac{\rho_1 v_1^2}{2g} = \frac{\gamma}{\gamma - 1}\frac{\rho_1}{\rho_2}p_2\left[1 - (1 - x)^{\frac{\gamma-1}{\gamma}}\right]$$

$$= \frac{\rho_1}{\rho_2}p_2\left(x - \frac{x^2}{2\gamma} + \frac{1 + \gamma}{6\gamma^2}x^3 - \cdots\right).$$

Now $x = (p_2 - p_1)/p_2$, is usually very small; its largest value in this research being about 0.001; also ρ_1/ρ_2 differs from unity by less than 0.001, and $\gamma = 1.408$. Hence the equation reduces to:

$$\frac{\rho_1 v_1^2}{2g} = p_2 x = p_2 - p_1,$$

which is true to less than 0.1 per cent. for the range of velocities employed. Or, changing the notation, we have the formula:

$$\frac{\rho v^2}{2g} + p = p_0,$$

in which p is the pressure in the unchecked stream, p_0 the pressure in the impact mouth of the tube.

To demonstrate experimentally that the pressure p_0 is equal to the total head pressure, the following plan was pursued. A tube two and one half inches in diameter by eight inches long was inserted into the side of a tunnel in which a partial vacuum was maintained by means of an exhaust fan. A nozzle one eighth of an

inch in diameter, held in the center of this tube, transmitted the impact of the inrushing air to one cup of the pressure gauge, the other being connected with the quiet part of the tunnel. The gauge reading was taken; then the nozzle was thrust through the inlet tube toward the outside of the tunnel, and finally clear outside. No appreciable change of pressure could be noticed, thus showing that an impact nozzle sustains a constant pressure at all points of a stream-line, and that this is equal to the total head pressure. Again, the impact pressure was read at five different fan speeds, the nozzle being held first in the quiet outer atmosphere, then at the center of the inlet tube. The readings agreed to less than one per cent., as exhibited by the subjoined table.

TABLE I.

Comparison of Total Head and Impact Pressure at Point of Greatest Velocity.

Fan Speed.	Total Head.	Impact Pressure at Center of Inlet.
rev. min.	mg. per sq. cm.	mg. per sq. cm.
150	540	538
170	680	678
200	850	847.5
225	1,000	997.5
245	1,100	1,108

If it could be shown with equal exactness that the side nozzle measures the static pressure of the fluid gliding over it, the theory of the pressure tube would be completely verified. It is easy to measure the actual pressure sustained by the side nozzle at any point of the stream, but not so easy to prove the observed pressure equal to the true static pressure at the point, except where the velocity is zero or a maximum. It is zero where the stream tube is very broad, say in the reservoir, and here, of course, $p = p_0$; it is a maximum where the air rushes freely through the inlet tube, and here the static pressure must, according to theory, be equal to the pressure inside the tunnel. In fact when a narrow static tube, held at the center of a large inlet, was connected with one cup of the gauge, while the other cup was joined by hose to the quiet part of the tunnel, no material difference of pressure was observed. Hence we may conclude that the observed pressure equals the true static

pressure for that particular velocity. The experiment was made at five different fan speeds with like result.

It thus appears that for all the velocities employed, which extended from five to thirty miles an hour, the impact nozzle sustains exactly the pressure p_0, the static nozzle the pressure p. Hence, when the two nozzles a, b, Fig. 1, are connected simultaneously with the pressure-gauge, as is regularly done in practice, their differential pressure $p_0 - p$, must be an exact measure of the quantity $\rho v^2/2g$, and therefore determines the true velocity of the air. Now, by equation (1), $\rho v^2 = 2g(p_0 - p)$; or, writing $k = 1\sqrt{2g/\rho}$, $\delta = p_0 - p$, the velocity is:

$$v = k\sqrt{\delta}. \qquad (2)$$

In practice the differential pressure, δ, is read directly from the instrument in milligrammes per square centimeter, and k is computed from the observed elements of density. Thus for dry air at 0° Centigrade and a megadyne per square centimeter pressure, $\rho = 1.2759$ grammes per liter, whence $k = \sqrt{2 \times 980.96 \div 1.2759} = 39.21$ for the latitude of Washington, and the velocity in centimeters per second is:

$$v = 39.21\sqrt{\delta}.$$

For air at any temperature, pressure and humidity (Minchin's Hydromechanics), the density is:

$$\rho = 0.4645\,(8p - 3f) \div 8T;$$

p being the barometric pressure in millimeters, f the aqueous vapor pressure, T the absolute temperature Centigrade. From this k can be computed for any observed atmospheric conditions.

As a further test of the accuracy of the pressure-tube anemometer, its readings were compared with those of another instrument, designed with a special view to precision rather than convenience. Ten observations of the wind speed were taken simultaneously with both anemometers, and their average values agreed to less than one per cent. This extra instrument, which is sometimes used to standardize others, may be called the balloon anemometer. A brief description of its plan and performance may be worth a passing notice.

A toy balloon is held at the center of the wind-tunnel, between the prongs of a fork, one prong being firmly moored to the wall by a thread, the other joined to a release string. When the string is jerked the fork spreads and the balloon floats along the wind-current without rotation. After drifting less than eight feet it has acquired the full velocity of the wind, as may be proved by projecting the balloon with like speed in still air and observing how far it moves before coming practically to rest.

To find the velocity of the balloon two thin pencils of light are thrown squarely across its path and the time of transit recorded by a camera designed for that purpose. The general plan of the apparatus is shown in Fig. 2.

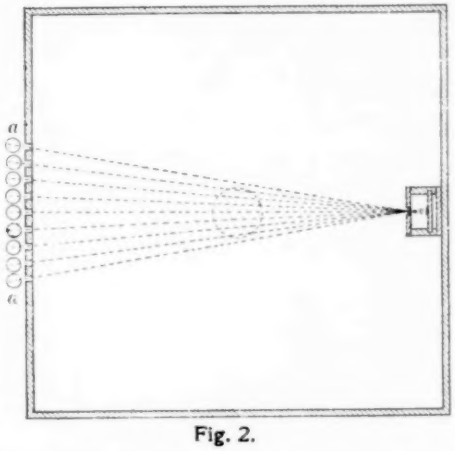

Fig. 2.

A bank of incandescent lamps *a*, *a*, shining through one-eighth-inch holes in the tunnel wall are brought to focus on a very sensitive photographic plate *b*. Ten feet farther along the tunnel a like thing occurs. The plates are clamped to a ten-foot board which is drawn by hand along grooves inside the camera box, the box itself being thirteen feet long. The images of the incandescent carbons trace, along each plate, fine straight lines which are momentarily interrupted by the passing balloon. The method is an adaptation of that used by the writer to measure the velocity of projectiles.[1]

To time the plate an electric tuning-fork interrupts one of the light beams 128 times a second, making the record a dotted line. After development the plate records are superposed and the number of vibrations counted from center to center of corresponding balloon breaks. This gives the time of transit of the balloon over the ten-foot stage, from center to center of the light screens, and thereby determines the wind velocity. The duration of transit is revealed

[1] "Resistance of the Air at Speeds below One Thousand Feet a Second," Philosophical Magazine, May, 1901.

accurately to less than one part in five hundred. For greater exactness the circuit is broken, and the fork vibrates freely during the balloon flight.

The accompanying table gives the data for ten observations at practically the same wind velocity. Measurements were not made at other speeds for lack of time.

TABLE II.

Comparison of Balloon Flight and Pressure-tube Readings. Barometer, 746.51 *mm.*; *Temperature*, 284° *C.*; *Saturation*, .50.

Pressure-Tube Readings.	Tuning-Fork Vibrations.	Velocity of Wind in Tunnel.	
		By P. Tube.	By Balloon.
mg. per sq. cm.		ft. sec.	ft. sec.
36.5	163.5	7.96	7.85
36	162.3	7.90	7.90
36	163.5	7.90	7.85
36.5	166.0	7.96	7.73
39.5	157.4	8.27	8.16
37.25	156.2	8.03	8.20
36	156.8	7.90	8.18
36.5	157.8	7.96	8.12
36.5	171.3	7.96	7.50
38.5	161.0	8.16	7.96
		mean 8.00	mean 7.95

The results show that the average air velocities, as determined by two such widely different instruments, agree to less than one per cent. The coincidence is rather too close to be representative; still the general agreement is such as to establish confidence in this form of pressure-tube anemometer, at least for the velocity at which the test was made. To assure ourselves that the instrument is reliable at other velocities, we may recall the experimental proof that each nozzle sustains exactly the theoretical pressure at all speeds up to thirty miles an hour.

Incidentally, in the course of other measurements, this anemometer has been compared with several instruments of a different type; notably two cup anemometers, a large and a small one, used by the United States Weather Bureau; a screw, and a cup anemometer employed at the Smithsonian Institution; and a Dines pressure-tube anemometer.

The following page from the laboratory note-book illustrates the calibration of the first instrument mentioned. It was placed in the tunnel beside the pressure-tube in such a way that neither interfered with the other. The time of fifty revolutions of the cups was indi-

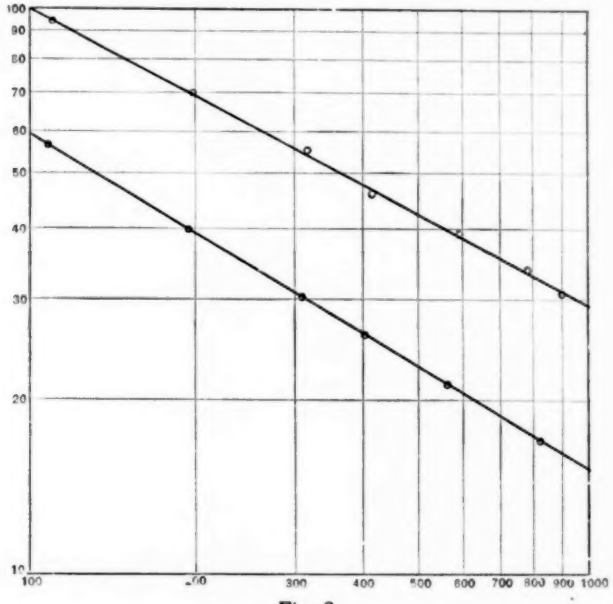

Fig. 3.

cated by an electric bell, and the interval between two or more ringings of the bell was measured by means of a stop watch reading to quarter seconds. The readings of the pressure gauge were simultaneously noted.

TABLE III.

Comparison of Standard Pressure-Tube Anemometer and Four Inch Cup Anemometer of the U. S. Weather Bureau. Barometer 30″.473, *thermometer* 3°.5 C.

Date.	Fan.	Pressure Tube.	Wind Speed.	4″ Cup Anemometer.	Wind Speed.
Feb. 20, '03	rev. min.	mg. sq. cm.	mi. hr.	sec. in 50 rev.	mi. hr.
2.15 p. m.	150	43	5.7	62.5	5.8
	200	83	7.9	44.5	7.9
	250	120	9.5	35.5	9.7
	300	180	11.7	29.0	11.7
	350	250	13.7	23.5	14.0
3.15 p. m.	400	295	15.0	22.0	14.9
	450	350	16.2	19.6	16.5

By comparing the wind velocities indicated by these two instruments it will be seen that they differ by less than two per cent. This is the case under favorable circumstances, when due attention is given to the working condition of the instruments; but, if they are not very carefully used, the discrepancy may exceed five to ten per cent. of the quantity measured.

In a similar way the calibration of the small Weather Bureau instrument, known as a kite anemometer, having cups one inch in diameter, and four inches from center to center, gave the following values.

TABLE IV.

Comparison of Standard Pressure-Tube Anemometer and One Inch Cup Anemometer of the United States Weather Bureau.

Date.	Fan.	Pressure-Tube.	Wind Speed.	Cup Anemometer.	Wind Speed.
Jan. 3, 1903.	rev. min.	mg. sq. cm.	mi. hr.	sec. in 500 rev.	mi. hr.
2.15 p. m.	200	110	9.53	121.25	9.45
	300	220	13.46	83	13.8
	400	370	17.46	65	17.55
	500	570	21.68	51	22.35
2.55 p. m.	600	830	26.14	41.5	27.4

The results obtained in standardizing the anemometers of the Smithsonian Institution are given diagrammatically to show the concordance of the observations. They were obtained in less than an hour for each instrument, without special pains; yet the data seem to harmonize very well, and to indicate that such instruments may give quite accurate readings when used with due care. One is a cup anemometer having cups one inch in diameter, and three and three eighths of an inch from center to center; the other is a Beck screw anemometer, two and five eighths of an inch in diameter, and having flat blades. The data are plotted on logarithmic cross-section paper for convenience, their relation being expressed by a straight line in such coördination.

Very interesting results were obtained from the pressure-tube anemometer invented by Mr. W. H. Dines, Fellow of the Royal Meteorological Society of England. This instrument is manufactured by Cassella, of London, and is extensively used, in one form or other, throughout Great Britain, and on the continent,

notably in the meteorological service. The essential difference in principle between the Dines pressure-tube and mine consists in the form of the static nozzle, which in his instrument is a punctured dome whose axis is at right angles to the steam-lines (see Fig. 4), while in mine it is a hole, or slot, so placed as not to deflect the steam-lines.

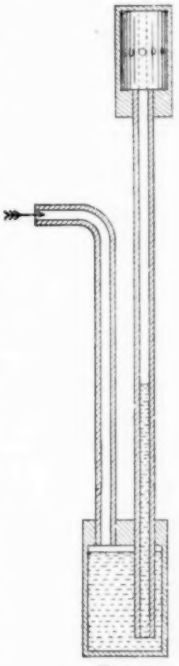

Fig. 4.

In order to compare the two instruments, the Dines static nozzle was employed as shown in Fig. 1, *b'*, and readings taken at several different fan speeds. Then similar readings were taken with the pressure tube shown in Fig. 1, *b*, under the same weather conditions. The results are presented in the accompanying table, and plotted to the same scale on logarithmic cross-section paper, as shown in Fig. 5.

The readings obtained with the Dines nozzle are approximately 1.46 times those found with the steam-line nozzle, for the velocities employed, which ranged from ten to twenty-five miles an hour. This excess is doubtless due to the suction in the rear of the punctured dome which constitutes his static nozzle. The dome, therefore, possesses the advantage of magnifying the readings of the instrument. Its hydrodynamic theory, however, is different from that of the stream-line nozzle, and its readings cannot be calculated from equation (2) of this paper.

TABLE V.

Comparison of Stream-Line Static Nozzle with Dines' Nozzle.

Fan Speed.	Pressure-Gauge Readings, $p_o - p$.		Ratio.
	With Stream-Line Nozzle.	With Dines' Nozzle.	
rev. min.	mg. sq. cm.	mg. sq. cm.	
200	205	305	1.50
250	310	460	1.48
300	440	640	1.46
350	590	850	1.44
400	760	1,080	1.42

It would be interesting to find a theoretical formula that would give, for Dines' instrument, the exact relation between the wind velocity and the differential pressure. For the range of velocities shown in Table V., the formula $v = k' \sqrt{\delta}$, will apparently give the true wind speed, if $1.46k'$ equals k of formula (2). But the comparison was too briefly made to be conclusive. Besides the formula thus obtained is only an approximate one, and does not free the instrument from an empirical constant.

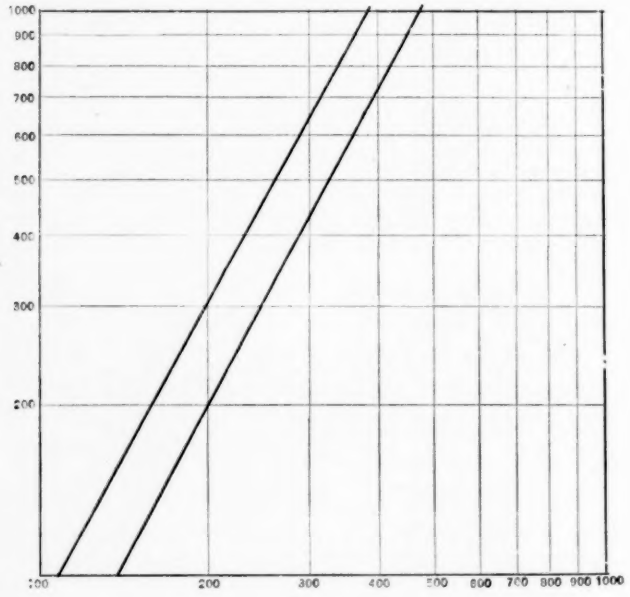

Fig. 5.

Various other forms of static nozzle have been used by different experimenters. The simplest of these is a straight open-ended tube, standing at right angles to the current. Another form consists of two thin circular discs, placed very near together, their axis perpendicular to the current; and in the modification used by Professor Nipher, a sheet of wire gauze fills the space between the discs, protruding slightly beyond their edges. Neither of these nozzles, however, has been compared with the one discussed in this paper.

I would refer the reader interested in the pressure-tube, as an instrument for measuring the velocity of liquids, to a very careful

and valuable paper by Mr. W. M. White, read before the Louisiana Engineering Society, May 13, 1901, and published in the Journal of Engineering Societies the following August. His methods of experimentation were quite different from the ones here outlined, but they established conclusively the theory of the pressure nozzle for water. Indeed this is as it should be, since the equation of motion is the same in both cases. Thus the two researches corroborate one another, if the excellent work of Mr. White can be said to need any confirmation.

In conclusion it is a pleasure to acknowledge my indebtedness to Mr. D. W. Taylor, Naval Constructor U. S. N., for his kind interest in this work. Indeed it was at his suggestion that the present investigation was taken up. He was making, for the government, a study of the ventilation of ships, with a view to determining the efficiency of different fans, and the most favorable working conditions of the ventilating system. He relied upon the Pitot tube to determine the flow of air at various points of the ventilating pipes, and, in October, 1902, expressed the wish to have its accuracy tested in a wind current of known velocity. This led to the device which I have described as the "balloon anemometer." The apparatus was made, and the observations were taken in November, 1902.

I have also to thank Professor C. F. Marvin, of the Weather Bureau, and Mr. C. M. Manly, of the Smithsonian Institution, for many valuable suggestions, for the use of a variety of anemometers, and for important literature and references pertaining to this work.

DEPARTMENT OF MECHANICS,
THE CATHOLIC UNIVERSITY OF AMERICA.

SECONDARY RADIOACTIVITY IN THE ELECTROLYSIS OF THORIUM SOLUTIONS.

BY GEORGE B. PEGRAM.

IT being a well-known fact, studied especially by Rutherford and Soddy,[1] that the surface of bodies in the air near thorium compounds becomes charged with a temporary radioactivity under the influence of the thorium emanation, and that this activity is much increased if the surface is negatively electrified, it seemed probable that some interesting results might be obtained in connection with the electrolysis of solutions containing salts of thorium. Accordingly solutions of thorium nitrate were subjected to electrolysis, the electrodes being of platinum, with the general result that the anode became more or less radioactive. This radioactivity was only temporary, in some cases decaying at the rate of half its intensity in eleven hours, in other cases much more rapidly. The secondary activity due to the emanation from dry thorium oxide decays at the rate of half its value in about eleven hours, according to Rutherford.

The thorium salt used in most of the experiments was a so-called chemically pure thorium nitrate, prepared from Brazilian monazite by E. de Haen, of Hanover, and sold by the firm of Eimer and Amend, of this city. When not otherwise specified this is the thorium nitrate referred to throughout the paper. From solutions of this salt an adherent anode deposit varying in amount from a slight tarnishing of the platinum to a reddish brown or sometimes brassy colored coating was obtained. This coating was found to be highly radioactive. Some comparisons between its activity and that of metallic uranium showed that mass for mass it was at least 2,000 times as active as the uranium. It has been examined chemically by Professor W. H. Pegram, of Trinity College, N. C., who

[1] Phil. Mag., Jan., 1900, p. 1, Feb., 1900, p. 161, Sept., 1902, Nov., 1902, Jan., 1903, April, 1903, May, 1903.

found it to be lead peroxide. The presence of this matter which is deposited on the anode proves the impurity of the thorium nitrate, but the phenomena are none the less interesting.

If a solution of very pure thorium nitrate is used for electrolysis very little matter is deposited on the anode and the resulting activity of the anode is comparatively small. Three samples of especially purified thorium nitrate were very kindly furnished me by some chemist friends, Mr. Neish of our department of chemistry, Professor Baskerville of the University of North Carolina, and Mr. Whitaker, chemist of the Welsbach Gas Mantle Company. From none of these could any visible deposit be obtained on the anode, yet after electrolysis for considerable periods the anodes showed a certain amount of activity, though small in comparison to that of the deposit from the de Haen thorium nitrate and differing from it in its rate of decay.

If to a solution of pure thorium nitrate is added a small amount of some salt such as lead nitrate or copper nitrate, a metallic kathode or oxide anode deposit may be obtained, and such deposits are found to be radioactive.

One might expect that the kathode in electrolysis would collect a larger amount of radioactivity than the anode, since in the air near thorium compounds a negatively charged body, or kathode, becomes more active than an uncharged or positively charged body. Such is not the case in the electrolysis of thorium solutions. A small amount of thorium hydrate is generally deposited on the kathode during the electrolysis, but the temporary activity so noticeable on the anode is not present on the kathode. Since the thorium hydrate deposit can only be removed from the kathode by such treatment as would also remove the secondary activity if present, the kathodes tested for radioactivity were simply washed after the electrolysis and their activity tested at intervals over a period of several days, during which time no change could be detected in their activity, from which it is inferred that the total activity was due to the constant activity of the thorium hydrate. If a very heavy current density is used in the electrolysis the thorium hydrate comes down at the kathode as a somewhat gelatinous precipitate, which is redissolved on standing by the nitric acid from the anode.

The present paper is then an account of a number of experiments in connection with the radioactivity of the peroxide of lead deposit from the de Haen thorium nitrate solutions, of the anodes used in pure thorium nitrate solutions, of the anode and kathode deposits from salts introduced into the thorium solutions, of the gases liberated by electrolysis, and some other related matters. A previous short paper on the subject was read by title at the Pittsburg meeting of the American Physical Society, June, 1902, and an abstract printed in Science, November 21, 1902, p. 825. Other experimenters who have subjected radioactive solutions to electrolysis have been Dorn,[1] who found that on the electrolysis of solutions both the anode and kathode become temporarily radioactive, the anode more strongly so; Sella, who electrolyzed thorium solutions and found both anode and kathode temporarily radioactive; and Marckwald, who has obtained a new radioactive substance resembling tellurium by the electrolysis of solutions of polonium.

Apparatus. — Electrolysis of the thorium solutions was generally performed in a straight-walled glass beaker, 6 cm. in diameter. For anodes hollow cylinders of platinum, 8 cm. long, 1 cm. diameter, were used, the open lower end resting on the bottom of the beaker, so that the inside of the cylinder should take no part in the action. A sheet platinum kathode, just fitting inside the beaker, was used, the anode being placed in the middle of the beaker. With 100 c.c. of solution 3.5 cm. of the anode was immersed, giving an anode surface of 11 cm^2. Current was obtained from the 115-volt lighting circuit, resistance being introduced into the circuit to regulate the amount of current. The current was measured by a voltmeter, graduated in thirtieths of a volt, across a 5-ohm resistance in series with the electrolytic cell. Low voltage current was obtained by shunting the electrolytic cell with a variable resistance. The voltage of the circuit was fairly constant, so that the variations of current were within the errors of measurement of the radioactivity. Immediately after use in the electrolysis the anode was always washed under a stream of water and quickly dried in a current of air before testing its radioactivity.

[1] Ueber die von radioaktiven Substanzen ausgegebenen emanation, Abh. d. naturf. Ges. Halle, 1900.

The radioactivity of the anode was measured by the conductivity it imparted to the air in a testing vessel, consisting of a metal cylinder 13 cm. high, 10 cm. in diameter, with removable lid, standing on a hard-rubber base. A brass rod, over which the cylindrical anode could be slipped, stood up through the middle of the base, below which it was connected to one pair of quadrants of an electrometer, the end of the rod below the hard-rubber base, and the short wire leading to the electrometer being surrounded by a grounded metal tube. This rod, and therefore the electrometer quadrants, was ordinarily grounded through a spring contact piece, but a slight pull on a string served to break the contact. The metal cylinder was connected to the negative pole of a battery of 200 dry cells (280 volts). When a radioactive anode is slipped over the central rod, the air in the vessel is ionized by the radiations and conduction takes place across the air column, thereby causing the electrometer needle to move at a rate proportional to the current. The voltage used was sufficient to utilize practically the total conductivity of the air column, and thus to obtain the maximum current. It has been shown [1] that under these conditions the current between the cylinder and central rod is very nearly proportional to radioactivity of the surface of the central rod.

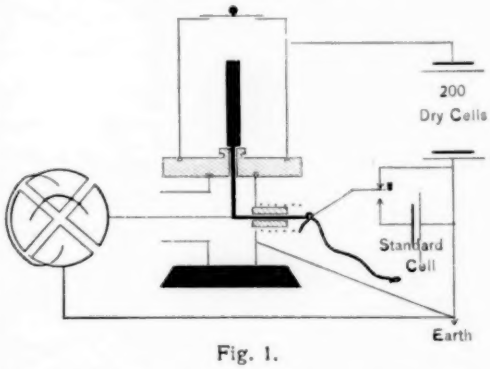

Fig. 1.

The electrometer used is of the Dolezalek [2] quartz fiber suspension type, constructed in the department shop. Amber insulation, which is excellent even in comparatively damp air, supports the quadrants. The needle is of hard rolled aluminum foil .001 inch thick, of the Maxwell pattern and 3.8 cm. long. The use of a single piece of aluminum has some obvious advantages over the double thickness gilt paper needle of Dolezalek, especially as to

[1] E. Rutherford, Phil. Mag., February, 1900.
[2] Zeit. fur Inst., December, p. 345, 1901.

lightness and permanence of form under varying hygrometric conditions, though the damping is not quite as great as with the paper needle. The needle was charged by being kept in connection through the suspending quartz fiber, which had been dipped in a solution of calcium chloride, with the negative pole of a battery of 100 dry cells (140 volts), and at this voltage usually gave a deflection of 360 mm. divisions on a scale 130 cm. away when a Clark cell was connected in between the quadrants. The Clark cell could be thrown in by simply pressing a key and was used not only for checking up the sensitiveness of the electrometer, but also for quickly bringing the needle to rest at zero after a reading, since the suspended system was not quite dead-beat. The electrometer has been used for weeks without changing its sensitiveness more than 2 or 3 out of 360 divisions. Sometimes the electrometer had to be moved and then its sensitiveness changed, so that all the measurements which are compared with one another are reduced to the same electrometer sensitiveness by applying the ratio of sensitiveness on the two occasions as determined by the Clark cell, which gives a result very nearly correct, though it is true that the changes of sensitiveness to difference of potential and of sensitiveness to quantity of charge do not necessarily take place in the same ratio. In some of the measurements it was necessary to decrease the sensitiveness of the electrometer very much, and to do this a small leyden jar, capacity .0005 microfarad, was connected to the electrometer and testing vessel. This decreased the rate of movement of the needle with a given current to .115 of its rate without the jar attached, so the capacity of the testing vessel and electrometer is $.627 \times 10^{-4}$ microfarad. With the usual sensitiveness of 360 divisions to 1.43 volts a rate of 1 division per second corresponds to a current of 2.5×10^{-13} ampere through the testing vessel. There was always a small leakage from the testing cylinder to the electrometer, usually amounting to .13 division per second. It was quite constant and has been allowed for in the results when appreciably affecting them.

The Radiations from the Anode Deposit. — The secondarily active deposit gives off radiations chiefly of the non-penetrating α^1 type, with a small proportion of a more penetrating type, as is shown by

[1] Rutherford, Phil. Mag., January, 1899.

the absorption of the radiations by different thicknesses of aluminum. At the same time a radioactive gas similar to the thorium emanation is given off, which remains in the air of the testing vessel, giving it considerable conductivity for a few minutes after the active anode is taken out of the testing vessel. The conductivity is proved to be due to a radioactive substance in the air inside the testing vessel by the fact that when the air is blown out of the testing vessel the conductivity immediately drops to zero.

The following measurements made on an anode made quite radioactive show the penetrating power of the radiations through aluminum. The aluminum was in the form of cylinders of foil fitting closely around the radioactive anode. When no foil was used the conductivity of the air column, with this particular anode, was too great to measure without changing the electrometer arrangements, so a cylinder of lead with a longitudinal slit to expose one fifteenth of the active surface, was put over the anode and the resulting electrometer rate multiplied by 15.

Anode with Radioactive Coating from the de Haen Thorium Nitrate.

No. Thicknesses of Al. Foil, .04 mm. Thick.	Electrometer Divisions per Sec.
0	4,000
1	417
2	41
4	24
7	19
11	18
15	12

The question whether or not the ratio of radioactivity to mass of the lead peroxide deposit was constant during all stages of the electrolysis was investigated by using successive anodes in the solution for definite periods of time, and testing their activity, after which the deposit was dissolved off and the loss of weight determined. The balance used was sensitive to $\frac{1}{10}$ mg., but this of course did not enable a very accurate measurement of the mass of the deposit to be made, since the total mass was so small.

The small Leyden jar was connected to the electrometer during the measurements. Without it the number of divisions per second above would have been nearly nine times as great, too fast a rate to measure. It appears from the variation in the ratio of the activity to

Solution, 15 *gm. thorium nitrate in* 100 *c.c. water. Current* .84 *ampere.*

Anode Used.	Mass of Deposit.	Elec. Div. per Sec.	Ratio Activity to Mass.
First 20 minutes.	2.8 mg.	60	1.00
Next 20 "	2.1	43	.94
" 60 "	2.0	100	2.3
" 20 "	.3	13	2.0

the mass that the more slowly the matter is deposited the greater its activity.

The Effect of Continued Electrolysis. — The following results show how the activity of the anode when used in a solution of the de Haen thorium nitrate rapidly decreases with the duration of the electrolysis, due chiefly to the decreasing amount of the brown deposit on the anode. The experiment consisted in putting in fresh anodes at intervals as the electrolysis proceeded for periods of five minutes and testing the resulting radioactivity of the anodes.

Solution of 40 gm. $Th(NO_3)_4$ in 400 c.c. water. Current .44 ampere.

Duration of Electrolysis.	Activity of Anode, Electrometer Divs. per Sec.
5 minutes.	12.50
15 "	(50.00)
5 "	11.55
35 "	
5 "	6.25
35 "	
5 "	2.70
75 "	
5 "	1.62
365 "	
5 "	1.25

Thus by 475 minutes' electrolysis with the current strength as above the power of the solution to deposit radioactive matter on the anode had decreased to one tenth of its original amount. No doubt if the solution could be kept perfectly neutral the lead peroxide would come down faster, for the presence of free nitric acid around the anode tends to keep it in solution. After long electrolysis, when there is no longer any visible amount of matter deposited on the anode, the solution reaches a condition in which the power of depositing radioactive matter on the anode suffers little or no further decrease. The amount of activity gained by an anode from such a solution is

comparable to that obtained from the purest thorium solutions, so that it represents perhaps a deposition of radioactive matter unmixed with any ordinary matter from the solution.

It seemed probable that the radiations from the thorium salt itself might be changed in quality or quantity after being in a solution subject to electrolysis. The oxide being more convenient to work with than the nitrate, 100 mg. thorium oxide was prepared from the nitrate by heating it to a low red heat, and also 100 mg. from nitrate from a solution that had undergone long electrolysis, until very little radioactive matter was being deposited on the anode. The activity of the two samples of oxide was practically the same in amount, but the oxide from the nitrate that had been in the solution seemed to give off a slightly smaller amount of the radioactive emanation.

Activity of Anode as Related to the Amount of Thorium Salt.— The activity of the anode depends, other conditions being the same, on the total mass of the thorium nitrate used in the solution, except when dilute solutions, less than 2 gm. to 100 c.c., are used. Solutions of 100 c.c. water with different amounts of thorium nitrate were made up and subjected to electrolysis with equal currents for equal times and the activity of the anodes tested.

Time of electrolysis in each case five minutes. Current .44 ampere.

Solution.	Activity of Anode, Electrometer Divs. per Sec.
1.25 gm. Th(NO) in 100 c.c. water.	1.85
2.50 " "	3.03
5.00 " "	6.45
10.00 " "	15.8
20.00 " "	31.3

With the exception of the first and weakest solution the activity increases pretty regularly with the amount of thorium salt in solution. Other experiments with dilute solutions showed considerable irregularity in the results.

If we keep the total mass of thorium nitrate constant, but vary the concentration of the solution by increasing the volume of water, not much difference is found in the activity of the anode. From 2.5 gm. thorium nitrate in 100 c.c. water, the anode gave 3.03

electrometer divisions per second, from 2.5 gm. in 200 c.c. water, the rate was 3.80 divisions per second.

The area of the anode in the solution, or in other words the current density at the anode, has considerable influence on the amount of activity. With 1 cm. of anode immersed in a certain solution, current .44 ampere, the activity was such as to give an electrometer rate of 2.70 divisions per second; with 7 cm. immersed in a similar solution the rate given was 4.16.

Activity as Related to Duration of Electrolysis and Current Strength. — As was expected the ratio of the activity of an anode to the time it has been used in electrolysis is nearly constant, when that time is not so long as to produce a noticeable effect on the power of the solution to deposit radioactive matter. Anodes were used in the electrolysis of a 500 c.c. thorium nitrate solution for various time intervals. The ratio of the activity to the time ran as follows:

Time of Electrolysis.	Ratio Activity of Anode to Time.
2 minutes.	.83
4 "	1.00
6 "	1.05
8 "	1.04
10 "	1.01
12 "	1.05
14 "	1.11
16 "	1.11

The relation between the current density and the amount of radioactivity on the anode is seen from the following results: The reason for the decreasing efficiency of the stronger currents is no doubt due to the nitric acid becoming so concentrated at the anode that the deposition of the brown coating was hindered to some extent. If very heavy current densities were used the brown deposit was sometimes redissolved.

Current Density at Anode, Amperes per cm^2.	Ratio Current Density to Activity.
.017	1.00
.037	.89
.085	.87
.132	.69

With a voltage across the electrolytic cell just below that required to liberate oxygen and hydrogen at the electrodes lead peroxide is

not deposited and the amount of radioactive matter of any kind deposited on the anode is small. From a solution of 5 gm. thorium nitrate no visible deposit was obtained in three hours, and the anode when tested showed no radioactivity.

Since thorium oxide is all the time giving off an emanation which causes secondary radioactivity, the following experiment was tried. 6.5 gm. thorium oxide was ground fine in a mortar and shaken up with 100 c.c. water. This water, with the thorium oxide still present in it, was subjected to electrolysis for twenty minutes, current .44 ampere. No radioactivity could be detected on either the anode or kathode. This is in accord with the results of Rutherford,[1] who finds that the emanating power of thorium, and therefore its power of imparting secondary activity to the surface of bodies in the neighborhood is at a maximum when it is in solution.

The Oxygen and Hydrogen from Electrolysis. — The oxygen and hydrogen liberated in the electrolysis of a solution of 20 gm. thorium nitrate in 300 c.c. water were collected separately in tubes above the electrodes and led at will into a testing cylinder, on the way passing through plugs of glass wool and a calcium chloride drying tube. The testing vessel consisted of a cylinder of brass 21 cm. long, 6 cm. in diameter, closed at each end with a plug of hard rubber. Through a hole in one plug the gas was led in from the drying tube, while a hole in the other allowed it to escape after traversing the length of the cylinder. A brass rod fixed to one of the hard-rubber plugs passed down the middle of the cylinder nearly to the other end. This rod was connected to the electrometer and the cylinder to a source of potential, usually 280 volts. The current between the cylinder and central rod, as given by the electrometer rate, was taken as a measure of the radioactivity of the gas.

The volume of the collecting and drying tubes and connections was 95 c.c., so with a current of .84 amp. through the solution, giving .106 c.c. hydrogen per second, it took the gas so long to get from the electrode to the testing vessel that the radioactive matter present had lost nearly all its activity, and the electrometer rate was very small. When the current was increased to3.2 amp., giving .403 c.c. hydrogen per second, and the hydrogen led into the

[1] Phil. Mag., April, 1903.

testing vessel, the electrometer rate became steady in about fifteen minutes at 2.40 divisions per second. As soon as the hydrogen tube was disconnected the current began to decrease in the manner indicated by the following figures.

Hydrogen from Electrolysis of Thorium Nitrate Solution in Testing Vessel.

Hydrogen flowing at rate of .403 c.c. per sec.,	2.40 elec. divs. per sec.
45 sec. after hydrogen flow was stopped,	1.60 " " " "
60 " later,	.80 " " " "
120 " "	.18 " " " "

Thus the rate of decrease of activity is about half in sixty seconds, or the same as that observed for the ordinary emanation from thorium, which is doubtless the radioactive matter present in the hydrogen.

When the oxygen was lead into the testing vessel, the same current flowing as above, giving .202 c.c. oxygen per second, a steady electrometer rate was attained in a few minutes. Results similar to those given above for hydrogen were obtained.

Oxygen from Electrolysis of Thorium Nitrate in Testing Vessel.

Oxygen flowing at the rate of .202 c.c. per second,	8.6 elec. divs. per sec.
20 sec. after oxygen flow was stopped,	7.5 " " " "
60 " later,	3.7
60 " "	1.9

The rate of decay is the same as in the case of the hydrogen. The amount of activity measured would have been less than half that in the case of the hydrogen even if the concentration of the radioactive emanation were the same in the two gases when first liberated, since it took the oxygen twice as long to get from the electrode to the testing vessel and the activity of the emanation was decreasing in the meanwhile.

To decide the question of whether the radioactive matter was present in the same contentration in the two gases when first coming up from the solution the current was reduced to half so as to give the same rate of flow of hydrogen as of oxygen in the experiment above, and the hydrogen again led into the testing vessel. Now the steady rate attained was 7.5 electrometer divisions, or 12 per cent. less than with the same rate of flow of oxygen. The current was then decreased to .84 amp. and the electrolysis continued.

After 3 hours the current was again increased to 3.2 amp. and the hydrogen led into the testing vessel. The electrometer rate became steady at 15.1 divisions per second, as against 24.0 divisions when the electrolysis was first begun. This decreasing amount of the radioactive emanation in the gas as the electrolysis proceeds is sufficient to account for the difference in the concentration of the radioactive matter in the two gases, since the electrolysis had been continued for thirty minutes between the time of the measurements on the oxygen and those on the hydrogen. It is therefore probable that the concentration of the radioactive matter in the two gases is the same when they first leave the solution.

Electrolysis of Very Pure Thorium Solutions. — A 100 c.c. solution of (5 g. of) especially purified thorium nitrate which I was able to obtain through the kindness of Professor Baskerville was subjected to electrolysis for 20 minutes with a current of .84 ampere. No deposit was visible on the anode or kathode. The anode when tested for radioactivity gave an electrometer rate of 6.2 divisions per second. It was left in the testing vessel and 12 hours later was found to have practically no radioactivity. This was rather surprising, as an anode from the de Haen thorium nitrate would have lost only a little over half its activity in this time. The solution was again subjected to electrolysis under the same conditions and the anode afterwards tested at intervals with the following results:

Anode from the Pure Thorium Nitrate Solution (Baskerville).

Immediately after the electrolysis.		5.4	electrometer divs. per sec.
75	minutes later.	2.5	" " " "
70	" "	1.1	" " " "
183	" "	.6	" " " "
121	" "	.3	" " " "

From this it appears that the rate of decay of the secondary radioactivity excited by thorium is not a definite quantity. The rate indicated above is about half in one hour, the rate heretofore observed half in about eleven hours. The same solution was again subjected to electrolysis and a repetition of the results obtained.

Deposition of Radioactive Metals and Oxides from Thorium Solutions. — As mentioned before, it appears that when any metal or oxide is deposited electrolytically from a solution which contains

thorium, such deposit possesses a temporary radioactivity. A few milligrams of copper nitrate were added to a solution of 5 gm. thorium nitrate in 100 c.c. water and a current sent through the solution, the current density being adjusted to give a bright adherent deposit of copper on the kathode. The voltage across the cell was 1.8. A clean but very thin coating of copper was obtained in five minutes and the activity of the copper coated kathode then tested.

Kathode Coated with Electrolytic Copper from a Thorium Solution.

5	minutes	after end of electrolysis.	2.62 electrometer divs. per sec.
3	"	later.	2.42
3	"	"	1.82
6	"	"	.83
2	"	"	.58
7	"	"	.14
200	"	"	.00

Here the rate of decay is even more rapid than in the case of the anodes from the pure thorium nitrate, though the source of the activity is the same thorium salt which gives an anode deposit, the activity of which decays at the rate of one half its value in eleven hours.

To get a deposit of oxide on the anode .1 gm. of lead nitrate was added to a 100 c.c. solution of 5 gm. thorium nitrate, and the solution subjected to electrolysis for 15 minutes with a voltage of 2.10 across the cell. A thin adherent coating of lead oxide was obtained on the anode. The following results show the amount of its radioactivity and its rate of decay.

Anode Coated with Lead Oxide Deposited from a Thorium Solution.

3	minutes	after electrolysis.	8.5 electrometer divs. per sec.
1	"	later.	6.8
3	"	"	4.0
4	"	"	2.5
7	"	"	1.8
32	"	"	.6

This is another example of rapid decay of the secondary activity from thorium.

Precipitates from Thorium Solutions. — Precipitates formed chemically in a solution with thorium nitrate present are radioactive. Silver chloride being a convenient precipitate to handle, tests were made

of the rate of decay of its activity. About .1 gm. silver nitrate was added to a solution of 5 gm. thorium nitrate in 100 c.c. water. After the silver had been in the solution ten minutes it was precipitated out with the requisite amount of HCl, excess of acid being avoided. The precipitate of silver chloride was filtered out, dried over a water-bath, transferred to a small metal disc, and this set on the top of the central rod in the usual testing vessel. Measurements were taken at intervals.

Silver Chloride Precipitated from a Solution Containing Thorium.

8.5	minutes after precipitation.	26.8 electrometer divs. per sec.
30	" later.	19.0
60	" "	11.0
95	" "	4.5

A further study of such radioactive precipitates will doubtless prove of interest. One experiment relating to the effect of the precipitation on the radioactive properties of the solution itself was tried. About 1 gm. silver nitrate was added to a solution of 5 gm. thorium nitrate in 100 c.c. water and then the silver precipitated out with HCl. The solution was immediately subjected to electrolysis for five minutes with a current of .84 amp. and the activity of the anode tested.

Anode from Solution of Thorium Nitrate in which Silver Chloride had been Precipitated.

180	seconds after end of electrolysis.	7.4 electrometer divs. per sec.
100	" later.	1.1
40	" "	.4
60	" "	.1
60	" "	.0

Then a similar solution of thorium nitrate without the addition of the silver salt and HCl was electrolyzed, after .5 c.c. HNO_3 had been added to make the amount of free acid in the solution at least as great as in the other solution. The activity of the anode after five minutes' electrolysis, current .84 ampere was such as to give an electrometer rate of 8.0 div. per. sec., with the usual rate of decay for the activity of the anode deposit from this de Haen thorium nitrate.

The rates of decrease with the time in intensity of the radiations from the various secondarily active substances obtained from thorium solutions is shown graphically in Fig. 2 by the curves, which are

plotted, as far as possible, so that the ordinates, representing intensities of radiation, are the same at the start, that is, immediately after the electrolysis or precipitation. Curve *I* is plotted for the radiation from the lead peroxide deposit, and it is to be noted that its intensity increases at first in the same way as has been observed by Rutherford for the radiation from bodies made secondarily active by the thorium emanation, then it decreases in a geometrical pro-

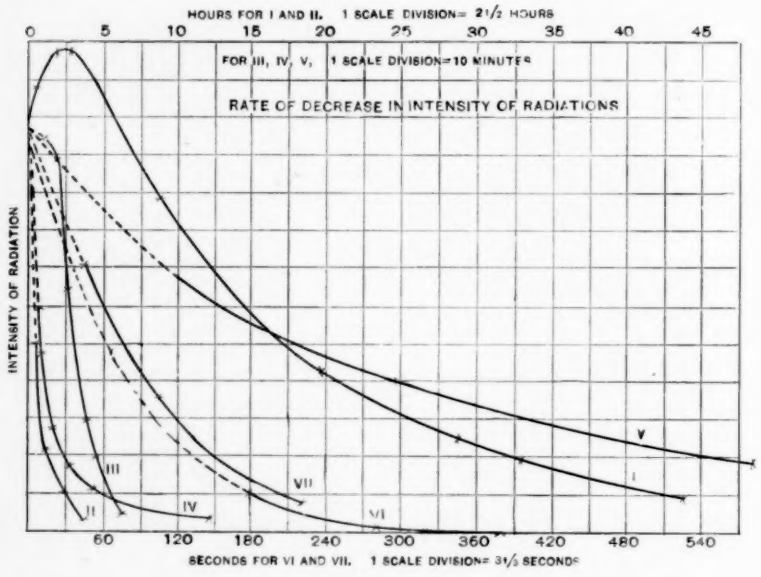
HOURS FOR I AND II. 1 SCALE DIVISION= 2½ HOURS
0 5 10 15 20 25 30 35 40 45
FOR III, IV, V, 1 SCALE DIVISION=10 MINUTES
RATE OF DECREASE IN INTENSITY OF RADIATIONS
INTENSITY OF RADIATION
II
III
IV
VII
VI
V
I
60 120 180 240 300 360 420 480 540
SECONDS FOR VI AND VII. 1 SCALE DIVISION= 3½ SECONDS

Fig. 2.

gression with the time, falling to half value in about eleven hours. If I_0 is the intensity at the beginning, and I the intensity at a time t later, we have $I = I_0 e^{-at}$, in which a may be determined from the fact that $I = \frac{1}{2} I_0$ when $t = 11$ hours. This decrease in approximately a geometrical progression with the time seems pretty general with secondary radioactivity. Curve *II* is plotted from the figures already given for the radiation from an anode after use in a very pure thorium nitrate solution, its decrease being about half in sixty minutes; *III* is for the radiation from copper deposited from a thorium nitrate solution, decrease half in seventy-five minutes; *IV* for that of lead oxide deposited similarly, decrease at first half in about sixty minutes; *V* for that of the silver chloride precipitated in

a thorium solution, decrease half in eighty minutes; *VI* for that of the anode used in such a solution after the silver was precipitated, decrease half in forty seconds; and *VII* for that of the active matter in the hydrogen from the electrolysis of a thorium nitrate solution, decrease half in one minute.

Summary. — In the electrolysis of solutions of the thorium nitrate prepared by de Haen, of Hanover, a deposit of lead peroxide is obtained on the anode, and this deposit is highly radioactive at first. The intensity of its radiations increases slightly for about two hours, then decreases, falling to half value in about eleven hours. The radiations consist chiefly, if not wholly, of the non-penetrating α type of radiations, with the exception that a small amount of radioactive gas similar to or identical with the thorium emanation is given off. The kathode has no secondarily active matter deposited on it.

The gases liberated in the electrolysis of a thorium nitrate solution are charged with radioactive matter, doubtless the ordinary emanation from thorium, which loses its activity at the rate of half in about one minute.

From solutions of the purest thorium nitrate obtainable no visible deposit is obtained on the anode, yet it is radioactive. The activity in this case decays rapidly, about half in one hour.

Anode and kathode deposits of oxides or metals from a solution containing thorium are radioactive. Their activity rapidly decreases, falling to half in a few minutes.

Precipitates thrown down chemically from solutions containing thorium nitrate are radioactive. Their activity also decays rapidly.

The secondary activity caused by thorium is not always the same in its rate of decay with time. The rate depends on how the secondarily active matter is separated from the thorium.

Coming to the question of the nature of the radioactivity shown by electrolytic and chemical deposits from thorium solutions, the lead peroxide, for example, two views are in general possible. First that the molecules or atoms of the lead peroxide have been so disturbed by being closely associated with the radioactive thorium that they now exhibit the properties of radioactive matter, or second, the radioactivity is a property of some kind of matter derived from the thorium which in some way becomes closely attached to the

lead peroxide. The second view of secondary radioactivity is strongly supported by most of the work relating to the question. Adopting it there is still the question of how the radioactive matter comes to be so closely connected to the lead peroxide as to be precipitated with it, is the connection an atomic, molecular, or simply mechanical one; and how does it come to exert an influence varying with circumstances on the rate of decay of activity of the radioactive matter? Experiments on the relation between the length of time the lead is present in the thorium solution and the amount of active matter it collects, also of the rate decay of the activity, easily suggest themselves and some of them at least would have been performed and the results given here, had not circumstances prevented the further use of the apparatus until the fall. We should expect no essential difference between the secondarily radioactive deposits obtained electrolytically and those obtained chemically, for the forces involved are of the same nature.

ON AN UNDESCRIBED FORM OF RADIATION.

FERNANDO SANFORD.

WHEN the terminals of the secondary winding of an induction coil are connected to the plates of an air condenser, the condenser plate connected to the kathode gives off a peculiar form of radiation into the condenser field whenever the current is interrupted in the primary coil. If a spark is allowed to pass between the discharging knobs of the secondary the radiation is given off from both condenser plates, but when no discharge occurs it is given off only from the kathode plate.

This radiation appears to the writer to be of the same nature as the ultra-violet radiation from a luminous body, and may be reflected, refracted and polarized as ordinary light. When it falls upon bodies placed in the condenser field these bodies become sources of a secondary radiation very similar in its properties to the primary radiation by which it is produced. This secondary radiation may, in the case of metals, at least, be given off both from the side of the body upon which the primary radiation from the kathode falls and from the side facing the anode. It may even be given off into an insulated hollow conductor placed in the condenser field.

In addition to this secondary radiation, the metals may reflect the original primary radiation.

Discovery. — This radiation was first observed by the present writer in 1892, and the observation was published in an article entitled "Some Experiments in Electric Photography" in the PHYSICAL REVIEW, Vol. II, p. 59, 1894. At that time some photographs were published which had been made by making a coin one of the plates of a condenser attached to the terminals of a very small induction coil, capable of giving a spark of only 2 or 3 mm. These photographs represented very clearly the relief figures of the coin, and one of them was made from a coin insulated

from the photographic plate by a thin sheet of mica. Attention was called to the fact that negatives had been thus made with the coin insulated from the photographic plate by paraffin, shellac and gutta percha, even through several millimeters of gutta percha, and the photographic effect was attributed to electro-magnetic waves sent off from the coin.

This explanation seems to have been unsatisfactory to all the physicists who have since written upon the subject, and the reduction of the silver salt has generally been attributed to the impingement upon the sensitized film of electrified air particles driven off from the charged coin or to negatively charged electrons projected from the coin.

In an article entitled "Patterns produced by Charged Conductors on Sensitive Plates" published in *Nature*, Vol. 55, p. 485, March 25, 1897, I called attention to the fact that negatives had been made of coins imbedded in the center of a block of paraffin 2 cm. thick when the paraffin block and coin were laid upon the sensitive plate in a box which was placed in an alternating condenser field and carefully insulated from the condenser plates by heavy sheets of plate glass. This seemed to me at the time conclusive evidence that the photographs were not made by streams of electrified matter sent off from the charged coin. The same article was accompanied by a coin photograph made with the coin carefully insulated from the condenser plates by panes of plate glass, and from the sensitive plate by a sheet of mica, but the photograph was not published with the article.

In an article in *Nature*, Vol. 65, p. 400, February 27, 1902, Sir William Crookes refers to the experiments just mentioned and says: "Electrons will easily pass through paraffin wax from the coin to the sensitive plate when the coin is connected with the negative pole of an induction coil, the other pole being connected with a metal plate placed below the wax block." This assumes that the photographic effects which had been attributed by the present writer to an electromagnetic radiation are really produced by the discharge of negatively charged electrons from the surface of the coin facing the sensitive plate. It also assumes that paraffin and mica are conductors, and that a discharge is constantly passing between the

plates of an alternating current condenser, no matter what the insulating material between them, since these photographs had been produced through all the best known insulators.

For the purpose of definitely settling the question whether the above described phenomena were due to a form of radiation or to a discharge of electrons, the experiments about to be described were carried on during the academic year just closed.

Method of Experimentation. — The experiments described below were made with a large induction coil and with a potential difference at the terminals of the secondary capable of giving a spark from 5 cm. to 15 cm. long when the condenser was connected across the terminals. An increase of this potential difference seemed to produce no other effect than to make the radiation more intense, and since this merely shortened the time of exposure of the sensitive plate while it made the necessary insulation more difficult, a potential difference capable of giving sparks from 6 cm. to 8 cm. long was generally used.

The primary current was broken by a rotating interrupter provided with a magnetic blow-out and driven by a motor at a speed which gave about ten interruptions a second, though the speed was varied through a considerable range without affecting the results. Various forms of condenser were used without any appreciable difference in the production of the radiation, but the condenser used in most of the experiments was made by pasting sheets of tin-foil about 10 cm. square on panes of glass about 30 cm. square and 5 mm. thick. In most of the experiments the plates were separated about 10 cm.

Various forms of camera, from a common black paper envelope to a tightly closed tin box, were used for holding the sensitive plate while in the condenser field, but the one most frequently used was made by fitting an old plate holder of a 5 x 7-camera to the top of a cigar box, so that the box could be closed by pushing in the hard rubber slide of the plate holder.

In exposing the sensitive plate to the radiation the condenser plate connected to the anode was usually supported horizontally with the tin foil below, the box camera was set upon the glass plate and the kathode plate was laid glass downward upon the box. The

box containing the sensitive plate was thus insulated from both condenser plates.

In some of the experiments hard rubber plates 6 mm. thick were used as insulators instead of the glass plates, but with no appreciable difference in the results.

Source of the Radiation. — In all the experiments previously described the photographs had been made while a discharge was taking place across a spark gap between the terminals of the secondary coil to which the condenser plates were attached. In the present experiments it was found that the radiation was more intense when no spark was allowed to pass, but that in this case it was given off only from the kathode plate, while in the other case it was given off from both plates. To compare the intensity of the radiation from the two condenser plates, two sensitive plates were cut from the same large plate, were placed back to back in a card-board box midway between the two sides of the box and held in position by wooden stops on both sides. The box was then placed on edge midway between two vertical condenser plates so that one sensitive plate would face each plate of the condenser. After the exposure, the plates were developed together for the same length of time. The plate facing the kathode was strongly blackened, while the one facing the anode remained clear. The blackening of the plate is accordingly due to something given off from the kathode.

When the kathode consists of a metal plate with only air between it and the sensitive plate there can be no question as to the source of the radiation, for in the case where the kathode is a coin the photograph shows clearly all the relief on the surface of the coin. When the kathode is a tin-foil sheet on glass or hard rubber there may be a question as to whether the radiation is derived from the metal or the insulating plate. Substances which are fairly opaque to the radiation when at a distance from the kathode may give it off profusely when in contact with the kathode. In experiments which were made to determine whether much of the radiation was stopped by the insulating plate, the hard rubber slide of the camera box was replaced by a slide of sheet zinc or copper which was itself made the kathode, and plates were exposed for the same length of time to the radiation from the bare zinc or copper and from the hard rubber

slide with the zinc or copper kathode laid upon it. In these cases the radiation was considerably more intense from the bare metal plate, but when an ordinary *papier maché* slide was used and tin foil placed upon it for a kathode the radiation was apparantly quite as intense as when a window was cut through the slide and the plate exposed directly to the tin foil. Since all of the insulating substances used are found to be more or less transparent to the radiation after it has left the kathode, it seems probable that the principal source of the radiation is in all cases at the face of the metal kathode.

Reflected and Secondary Radiation. — When the radiation from the condenser kathode falls upon other bodies a secondary radiation is given off from these bodies, and in the case of metals, at least, a

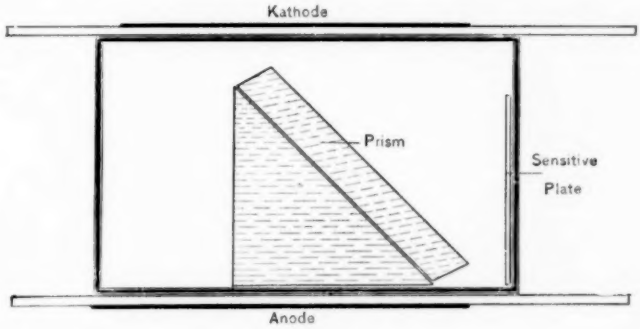

Fig. 1.

part of the primary radiation is reflected. In the case of non-metallic bodies the secondary radiation seems to be given off only from the face upon which the primary radiation falls, while in the case of metals it is given off from both sides.

For the purpose of comparing the secondary and reflected radiation from different surfaces the arrangement shown in Fig. 1 was used. The sensitive plate was set on edge at one end of the box camera already described. In this position it was found not to be affected by the primary radiation from the kathode. In front of it was a triangular wooden block lying on one side and having one face inclined at an angle of 45° to both the condenser kathode and sensitive plate, A triangular glass prism was laid upon this face of the block dividing it into two halves facing the vertical halves of the sensitive plate at the end of the box. The face of the block exposed

on each side of the prism was about 4 cm. by .7 cm. The glass prism served to shield a part of each end of the plate from the radiation from the opposite side of the prism. Negatives exposed in this way showed four separate areas rather clearly marked off from each other. Fig. 2 is from a negative made with a white visiting card on one side of the prism and a strip of red glass on the other side. As the picture here given is a positive the lighter parts are those upon which the radiation was more intense. The black triangle at the bottom shows the part of the plate shielded from both sources of radiation by the end of the prism, the light triangular area above this shows the region illuminated by both sources and the medium dark areas on each side were each illuminated by a single source. The shadows on each side were cast by pins which has been stuck in the sides of the box to hold the plate in position. It will be seen that the radiation was of nearly the same intensity from each source, though a little stronger from the glass. Figure 3 shows a similar photograph made with the wooden block bare on one side of the prism and the white card laid upon the other side. In this case the intensity of the radiation from the card is much greater than from the wood, but that from the wood was strong enough to show a distinct shadow of the pin.

Fig. 2.

Fig. 3.

In both photographs, as in all others made in this way, the dividing lines between the different areas are very sharp, and no sign

of a penumbra can be observed. Since the source of radiation was in each case about 4 cm. wide, it is hard to explain this lack of penumbra. Apparently this secondary radiation is given off approximately normal to the surface of emission.

When a strip of sheet metal of any kind was placed on one side of the prism the phenomenon became more complex. Fig. 4 is a photograph made from a strip of sheet aluminum on one side and the white card on the other side. In this case another rather clearly marked area appeared, as it did whenever any metal was used. This area, while showing clearly in the negative, is not brought out well in the photograph, and I have accordingly outlined it with dotted lines.

Fig. 4.

The metals accordingly seem to give off two kinds of radiation when the primary radiation falls upon them. One of these kinds gives the same sharply bounded shadow and inclined at the same angle as in the case of the secondary radiation from non-metallic bodies, while the other form bends much farther around the edge of the prism, as if it were sent off diffusely from all parts of the plate, though the gradual shading off in intensity which this should give is not perceptible in the negatives. Thus in the photograph given *A* is the area receiving both kinds of radiation from the aluminum, *B* the area receiving both kinds from the aluminum and the single kind from the card, *C*, the area receiving only the one kind from the aluminum and the radiation from the card, and *D* the area receiving radiation from the card only.

Presumably one of these forms of radiation is the primary radiation from the kathode reflected from the metal plate and the other is a secondary radiation induced at the surface upon which the primary radiation falls. Since non-metallic bodies give but the one kind, they are either reflectors of the primary radiation or sources of secondary radiation, but not both.

The question as to which form of radiation from the metal surface is reflected and which is a secondary radiation was apparently settled as follows: Previous experiments had shown that metals, as coins, when placed in the alternating condenser field give off a radiation on the side opposite the kathode, hence this cannot be reflected radiation. To compare this radiation with the two kinds given off from the kathode side of the plate, the glass prism was fastened with wax to the lower side of a copper plate which was inclined at an angle of 45° with the kathode and the sensitive plate, which was laid on the bottom of the box. A white card was pasted to the copper plate on one side of the prism while the other side was left bare. The photographic plate was thus exposed only to the radiation from the side of the plate opposite to the kathode. The negative showed the same sharp shadows on both sides of the prism that were so plainly seen in Figs. 2 and 3, and indicated that the secondary radiation given off by the metal was the same as that given off by non-metallic substances. This experiment was repeated with a zinc plate on one side of the prism and a white card on the other side, and the result was the same as with the copper plate. This agrees with the well known fact that metals are much better reflectors for all forms of radiation than are the non-metallic bodies.

The relative intensities of the reflected and the secondary radiations from the kathode side of the metal plates was not accurately measured. In general, the plate seemed more than twice as dark where the two kinds of radiation fell upon it as it did where it was exposed only to the reflected radiation. All metals did not seem alike in this respect, but no quantitative comparisons were made. Gold leaf on glass seemed a proportionally better reflector than any of the other metals tested. No appreciable difference in the total radiation from two strips of the same metal could be observed when one was highly polished and the other considerably tarnished.

A number of non-metallic substances were tested with reference to their power of emitting the secondary radiation. Transparent glass was the poorest radiator tested, and after it in their order came wood, white card, red glass and mica. The difference in the radiative power of wood and white card is shown in Fig. 3.

The metals, in addition to their reflective power, are much better radiators than the non-metallic bodies. Very little difference could be observed in the radiative power of different metals. The metals tested were aluminum, copper, gold, iron, tin, zinc and an amalgam mirror on glass. In the latter case the radiation had to pass through the glass, but it was apparently as strong as from the other metals. A lacquered iron ferrotype plate was compared with a piece of the same plate from which the lacquer had been polished off, and was appreciably poorer as a radiator than the polished iron. The difference in the quality of the secondary radiation from different metals will be discussed later.

The secondary radiation from the anode side of a metal plate is much weaker than from the kathode side. This was shown by supporting a plate of copper in the camera box at an angle of 45° to the condenser plates and placing one sensitive plate on the bottom of the box and standing the other upright at the end of the box, both at the same distance from and inclined at the same angle to the copper plate. The result showed that the radiation from the kathode side was very much the stronger. The radiation from the anode side may, however, be strong enough to affect a photographic plate very quickly if the plate is placed near it.

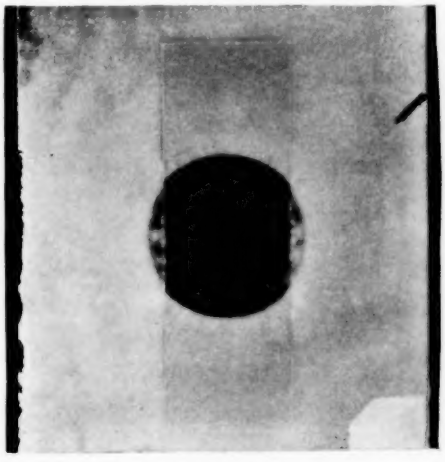

Fig. 5.

Fig. 5 is a photograph made froma coin resting on a glass microscope slide which was laid upon the sensitive plate in the box camera and exposed to the primary radiation from the kathode. In this case, as in all the coin photographs made in this way, the raised parts of the coin are lighter, whereas if the radiation had penetrated the coin they should have been darker. This particular photograph also shows a distinction between the primary and secondary radiation.

The area outside the coin is illuminated by the primary radiation from the kathode. It will be seen that the glass slide is very transparent to this radiation. The coin projects beyond the glass on both sides, and the coin photograph below these parts is much lighter than under the glass showing that the secondary radiation from the coin was very considerably absorbed by the glass. The glass is accordingly more opaque to the secondary radiation from gold than to the primary radiation from the kathode. Similar differences will be shown later to hold between the secondary radiations from different metals.

Secondary Radiation in a Closed Hollow Conductor. — One interesting fact in regard to the secondary radiation from metals is that it may be given off inside a closed hollow conductor. Fig. 6 is a photograph made on a plate which was placed in a tightly closed tin box and set inside the camera box. A piece of milk-glass thermometer scale was laid on the film of the sensitive plate. The plate was exposed in the condenser field for five minutes. It had been previously observed that the milk-glass scale was partly transparent to the radiation, while the ink of the scale was very opaque. This is shown in the photograph. The fact that the radiation penetrated the glass shows that the photograph was not due to the reducing effect of electrons from the sides of the box or the enclosed air.

Fig. 6.

Radiation from the Human Body. — When a sensitive plate was enclosed in a plate holder with hard rubber slides and laid in the condenser field and the hand was held upon the slide while the coil was being worked, the plate when developed showed a very black negative of the hand, showing that a very strong radiation was given out from the hand which was capable of penetrating hard rubber.

Regular Reflection. — Some attempts were made to focus the primary radiation by means of concave reflectors, but did not succeed. All the photographs which show a reflection of the primary radiation indicate that the reflection is largely diffuse. The fact that the best reflectors also give off a strong secondary radiation makes

it difficult to determine to what extent the primary radiation is regularly reflected. In many of the experiments where a slit was used between the secondary radiators and the sensitive plate, images of the slit were formed by reflection from the cover of the box, especially when this cover was of red glass, as it was in some of the experiments. This showed conclusively that there was some regular reflection of either the primary or secondary radiation. This part of the subject needs much more investigation than I have yet been able to give it.

Refraction. — Many experiments were performed on refraction with glass prisms and with hollow glass prisms filled with water, chloroform and other liquids. In general, these experiments were performed on the secondary and reflected radiation. Usually a strip of metal less than a centimeter wide was placed in the box at an angle of 45° with the kathode and a screen with a narrow vertical slit was placed about 5 cm. from the metal strip and between it and the sensitive plate, which was usually about 5 cm. from the slit. Sometimes two slits in line were used in order to secure the parallelism of the rays through the prism. With all the arrangements devised there were two images of the slit formed on the photographic plate. These images were always parallel, and were separated by greater or less distances according to the distance of the slit from the sensitive plate, being about 2 mm. or 3 mm. apart when the plate was 5 cm. from the slit. I am unable to explain the cause of this separation. The two images seem to be due to the two kinds of radiation, since when the metal strip was itself made the kathode only one image was formed. In some cases, also, the refraction was different for the two.

When a prism cut from a small spectacle lens was placed across a part of the slit the radiation which passed through the prism was refracted about four fifths as much as red light under the same conditions. When a flint-glass prism having a refracting angle of 24° was used the refractive index of the radiation from aluminum was found to be about $n = 1.7$. This is only a rough approximation.

When a hollow glass prism having a refracting angle of 14° was filled with water and placed before the slit it was seen that the two images of the slit were not equally displaced. One of the kinds of

radiation seemed to have a refractive index of about $n = 1.4$ and the other of $n = 1.1$. When filled with chloroform both refractive indices were nearly the same and were about $n = 1.5$. Later, it was found that the refractive index of the radiation may depend upon the nature of the metal which serves as its source. Several different metals were used as reflectors and the displacement of the slit images by a small crown-glass prism was determined, keeping all the conditions fixed except as to the metal reflector. In general, one image of the slit was displaced by about the same amount in all the experiments, while the other image was displaced more for some metals than for others. It was assumed that the image whose displacement was constant was due to the reflected primary radiation while the other was due to the secondary radiation sent off from the metal. When the photographic plate was at such a distance from the slit that the one image was displaced by about 2 mm., the other image was displaced as follows: For gold, 1.7 mm., tin, 2.8 mm., copper, 3 mm., aluminum, 4 mm. The displacement of the image produced by sunlight under the same conditions was 1.9 mm. A similar series of comparisons were afterward made with a flint-glass prism, and the differences in the refraction of the radiation from different metals were inappreciable. Differences in absorption corresponding with these differences in refraction are described later.

Total Reflection. — The slit images as described above were totally reflected by a right-angled prism so that they were formed at the side of the box, instead of directly in front of the slit. The radiation was also totally reflected when it was allowed to enter the base of a 50° prism and strike upon the face at an angle of 25°.

Polarization. — That the secondary and reflected radiation is not polarized when it leaves a copper plate was shown by passing it through two Nicols before it fell upon the photographic plate. The Nicols were placed side by side in the path of the radiation, but had their optic axes at right angles to each other. They seemed to transmit the radiation equally well.

The Nicols were then placed end to end with their optic axes parallel, and a plate was exposed for ten minutes to the radiation which might pass through them. Their axes were then crossed and

another plate cut from the same large plate was exposed for the same length of time, after which the two plates were developed together. The parallel Nicols allowed sufficient radiation to pass to give a very distinct image of their cross section, while the crossed Nicols quenched it completely.

Diffraction. — Many attempts were made to produce a diffraction spectrum by passing the radiation through gratings ruled on glass and by reflecting it from a metal grating. In the case of the glass gratings no diffraction could be observed. In one negative where an attempt was made to reflect an image of a slit from a Rowland grating a series of dark bands which looked like multiple images of the slit were formed on the plate but the plate was so strongly blackened by the secondary radiation and the diffusely reflected primary radiation that these images were not very distinct, and in several other negatives made as nearly as possible in the same way they could not be seen at all.

Attempts to produce interference bands by passing the radiation through a gypsum wedge between crossed Nicols were also unsuccessful.

Fluorescence. — When a tube of barium platinocyanide is placed in a strong alternating condenser field in a dark room the fluorescence of the tube may be plainly visible. To determine whether this florescence was due to the radiation in the condenser field, or to other causes, a glass tube of the cyanide was placed in the camera box near an upright sensitive plate and was exposed for several minutes to the radiation. No blackening due to fluorescence was visible when the plate was developed. A thin glass cell was then filled with the cyanide and placed directly against the sensitive plate and exposed to the radiation from a copper plate for several minutes. The salt served to screen the plate entirely from the action of the radiation, but showed no sign of fluorescence.

Absorption. — Both solids and liquids differ greatly in their transparency for both the primary and secondary radiation, and the same substance may differ in transparency for the secondary radiation from different sources. Most of the tests on transparency were made with the secondary and reflected radiation from a copper plate, though many were made with the direct radiation from copper, zinc and tin.

Of the solids tested, rock salt, quartz and glass, are the most transparent and their transparency is apparently in the order given above. Calcite is also very transparent, as is gypsum. Mica is much less transparent than glass and paraffin still more opaque. Colored minerals are usually less transparent than uncolored ones. Amethyst is fairly transparent, uranium glass less so, garnet still less and ruby and tourmaline very opaque. Ordinary red glass is very opaque, while the blue glass used in chemical laboratories is quite transparent. Cinnabar is fairly transparent, proustit is more opaque and zincite and rutil still more so. The metals are very opaque. A single layer of gold leaf or aluminum leaf seems to shut off the radiation completely. True, these give off a secondary radiation on the side opposite the kathode, but this is so weak as compared with the primary radiation which excites it, that if the photographic plate is placed at some distance from them they cast a complete shadow upon it. Some kinds of printing ink are very opaque. An aluminum advertising card laid upon a sensitive plate with the printed side next to the film, gave a negative on which the printing plainly appeared. The same phenomenon was shown in the photograph given in Fig. 6, where the printed scale is much more opaque than the milk-glass strip.

The metallic film deposited in a photographic negative is very opaque to the radiation, so that several positives were printed from negatives by its use. These positives differ but little, if at all, from those printed by sunlight. The intensity of the radiation was not great enough to print on sensitized paper, but the positives were made on glass plates which were exposed through the negative for several minutes and then developed.

In partially transparent substances, as in glass and mica, the absorption does not seem to be proportional to the thickness of the absorbing sheet. A glass cube 5 cm. thick showed but little more absorption than a glass microscope slide lying beside it. Several layers of mica sheets showed but little more absorption than a single sheet.

Absorption in Liquids. — The absorption of many liquids for the secondary and reflected radiation from a copper plate was tested by placing the liquids in a glass absorption cell about 1 cm. thick, which

was placed directly before the sensitive plate. In general the liquids were compared with water by using a double absorption cell with water in one compartment and the liquid to be compared in the other. Many of the liquids tested seemed perfectly transparent, so that no more absorption could be observed when the cell was filled than when it was empty. In fact, in some cases the filled cell seemed more transparent than the empty cell. This was especially the case in a solution of salt and water. Probably in these cases the liquids and the glass had very nearly the same refractive index for the radiation, so that there was less reflection than in other cases.

The liquids tested may be classed under the three heads, transparent, partly transparent and opaque. They are given below in this arrangement:

Transparent.	Partly Transparent.	Opaque.
Water.	Potassic bromide in H_2O.	Carbon disulphide.
Absolute alcohol.	Nickel sulphate " "	Aniline.
Ether.	Copper " " "	Red ink (Carter's).
Chloroform.	Lead nitrate " "	Iodine in H_2O + KBr.
Amyl alcohol.	Phenol pthyaline, red.	Iodine in alcohol.
Carbon tetrachloride.		Iodine in ether.
Ethyl acetate.		Potassic di-chromate.
Glycerine.		Nitro phenol in H_2O.
Turpentine oil.		Copper chloride in H_2O.
Toluene.		Copper chloride in HCl.
Iodine in chloroform.		Ferrous sulphate solution.
Ammonia.		$PtCl_4$ solution.
Sodium acetate solution.		Potassic ferricyanide solution.
Mercuric chloride solution.		Potassic ferrocyanide "
Cadmium nitrate "		
Ammonium sulphate "		
Potassic hydrate "		
Calcium chloride "		
Phenol pthyaline, solution.		

As in the case of light, the absorption of a substance in solution depends upon the nature of the compound formed in the solution rather than upon the nature of the dissolved substance or the solvent. Thus all the yellow solutions of iodine, though in transparent solvents, were quite opaque, while the red solution in chloroform was quite transparent. The colorless solution of phenol

pthyaline was quite transparent, while the red solution produced by adding a few drops of potassic hydrate solution was very opaque, though the potassic hydrate solution is itself very transparent. The difference in the photographic transparency of these solutions was greater than for light, as was shown by exposing one plate to light and another to the radiation through a double cell containing one solution in each side. The same difference was observed in other solutions, while carbon disulphide, which was optically perfectly transparent was very opaque to the radiation.

In some cases the absorption was very great for solutions of low concentration. A solution of platinum tetrachloride containing not more than one part by weight in two million parts of water was still appreciably opaque to the radiation from copper.

Differences in Secondary Radiation from Different Bodies. — For comparing the absorption of the secondary radiation from different substances the block inclined at an angle of 45° to the kathode and to the sensitive plate was used. A slit was sawn in the middle of the block and a triangular piece of red glass was set into it so that it stood with its vertical edge against the sensitive plate and prevented the radiation from one side of the glass from striking the other side of the plate. When different substances were laid upon the block on opposite sides of the glass, their radiations were kept separate on the plate. A glass absorption cell about 1 cm. thick and with faces about 5 cm. square was placed against the photographic plate, the block was moved so that the vertical edge of the glass partition came against the middle of one face of the cell and strips of the substances whose radiations were to be compared were laid upon the block. When the plate was developed after the exposure the photograph secured was as in Fig. 7. The dark vertical strip in the center shows the position of the glass partition. The light strip on each

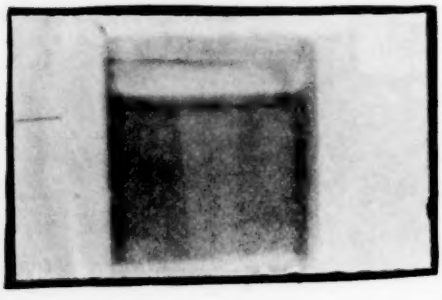

Fig. 7.

side of the center shows where the radiation from each side of the screen had lapped over onto the other side, and the darker strips outside of these show where each kind of radiation was absorbed by the liquid in the cell. In this particular photograph the absorption was nearly the same on both sides. The light horizontal strip above the liquid shows how much of the absorption was due to the glass.

The first comparisons were made in this way with a cell filled with a dilute solution of Carter's vermillion ink, which had been found very opaque to the radiation from copper. The substances used as sources of secondary radiation were wood, white paper, sealing wax, copper, tin, zinc, iron, aluminum and gold. The secondary radiation from the wood, paper and sealing wax was very completely absorbed. In the case of the radiations given off by the metals the absorption was greatest for the gold leaf and least for the iron. The differences between the other metals were inappreciable.

When the cell was filled with carbon disulphide the radiation from iron was absorbed least, that from aluminum came next, while that from copper, zinc, tin and a glass-amalgam mirror were about equally absorbed.

A dilute solution of $PtCl_4$ was found to absorb the radiation from the different metals in the following order: zinc or aluminum, copper, iron, tin.

A piece of window glass was more transparent to the radiation from iron than from copper, and apparently slightly more transparent to the radiation from aluminum than from iron.

As is well known to any one who has attempted photometric work, the above comparisons are necessarily very rough, as no photographic method will enable one to measure accurately the relative intensities of two sources of illumination; but it is believed that they are in the right order.

Probable Nature of the Radiation.— As will be seen from the foregoing discussion, this radiation differs very materially in its properties from ordinary kathode radiation and not less so from the Roentgen radiation and the radiations so far described from the radioactive substances. It seems to approach much more nearly in its properties to ordinary ultra-violet light than do any of the others.

In fact, the most probable guess as to its wave-length would seem to place it in the ultra-violet only a short distance beyond the visible spectrum. Since alcohol is very transparent to it, while it is absorbed very strongly by carbon disulphide, its wave-length would seem to lie between 250 $\mu\mu$ and 400 $\mu\mu$, and the fact that flint glass is very transparent to it would make it seem that its wave-length must be greater than 330 $\mu\mu$. In all probability, the wave-length depends to some extent upon the character of the radiating surface. Thus the radiation from iron, being less absorbed by carbon disulphide than that from other metals tested, would seem to consist, in part at least, of greater wave-lengths than the others, probably about 400 $\mu\mu$. It seems probable that waves of different length are sent off from the same metal. I do not yet despair of being able to measure the wave-length of the radiation from the different metals, though I have not yet been able to produce any interference effects. Of course, the possibility remains that the wave trains are too short to produce interference at sensible distances.

It has long been known that ultra-violet light may discharge negatively electrified metals, and it does not seem improbable that negatively electrified surfaces when suddenly charged and discharged may be capable of setting up a radiation of the same wave-length as that which is capable of causing their discharge. This makes it seem worth while to determine the particular wave-lengths which are capable of discharging different metals when negatively electrified.

Theory of the Radiation.— It is well known that if the experiments described had been carried on in the field of a condenser from which the air had been exhausted, the ordinary kathode radiation would have passed across from the kathode plate to the anode. Since this kathode radiation is believed to consist of negatively charged electrons driven off from the kathode, it seems probable that the same electrons will be sent off when the space between the condenser plates is filled with air. In the latter case, however, they would be stopped by colliding with air molecules before they had traveled far from the kathode, while in the former case they are stopped only by the anode or the walls of the containing vessel.

These electrons, when striking against the walls of the tube or against a metal plate contained in the tube, give rise to the Roentgen radiation, and it seems probable that their collisions with the gas molecules in the condenser give rise to the radiation described in this paper.

STANFORD UNIVERSITY,
June, 1903.

ON THE THEORY OF THE COMPOUND PENDULUM.

BY ROBERT R. TATNALL.

THE theory of the compound pendulum, considered as a rigid body making small oscillations, under gravity, about a fixed horizontal axis, is to be found in all treatises on dynamics; and the precautions and corrections which are necessary when such a body is to be used as an instrument of precision, have been carefully studied by Bessel, Helmert and many other investigators.

There is one aspect of the question, however, which, so far as I can discover, has nearly, if not quite, escaped notice, but which, by reason of its bearing upon the theory of the reversible pendulum, is not lacking in interest or importance. I refer to the case in which a definite portion of the oscillating body is movable with respect to the remaining portion. Or, in other words, the form of pendulum under discussion is that composed of two rigid masses, one of which carries the knife-edge, while the other may slide over the first, and be clamped to it at any desired point. The general problem, then, is to find how the period of the pendulum is affected by any specified change in the position of the movable mass.

I shall here consider only the special case in which the movable mass has its center of gravity always in the plane through the axis of rotation and the center of gravity of the first mass. This condition is realized in Kater's pendulum; it is also very approximately realized in clock pendulums regulated by a screw beneath the bob, but the latter are usually suspended by a flat spring, instead of a knife-edge, and so do not oscillate about a fixed axis.

While attempting to adjust a Kater pendulum belonging to this laboratory, I was surprised to find that when the movable bob was not far below the knife-edge, *the period of the pendulum was shortened by lowering the bob, and lengthened by raising it.* On further examination it became evident that if the bob, placed initially at a

point near the knife-edge, was moved gradually downward, the period of the pendulum decreased until a minimum value was reached, after which it increased again.

It is quite possible that this fact is already well known, but I have been thus far unable to find any discussion of it in the literature of dynamics. Probably no little confusion has resulted from the following erroneous, though somewhat popular, presentation of the compound pendulum idea. Suppose that a simple pendulum (bullet and thread), is suspended immediately in front of a compound pendulum, and from the same axis. If the length of the thread be equal to the distance from the axis to the center of oscillation of the compound pendulum, the latter will be unchanged in period by having the bullet attached to it. If the simple pendulum be lengthened, it will, by reason of increased period, and consequent tendency to lag in phase, retard the motion of the compound pendulum, when attached thereto, and so lengthen its period. On similar grounds it is frequently asserted that if the thread be shortened, instead of lengthened, the effect of consolidation will be to accelerate the compound pendulum, and shorten its period. This is equivalent to the assertion that if a compound pendulum be regarded as made up of an infinitude of material particles, then each particle which is farther from the axis than the center of oscillation will have a tendency to retard the pendulum, and so increase its period, while each particle which is nearer the axis than the center of oscillation will tend to diminish the period. That the last statement is erroneous may be easily seen. For if a particle of matter be added either at the axis, or at the center of oscillation, the period is not affected. But if it be added at any intermediate point, the period will be changed. Hence, if a particle be moved continuously from the axis to the center of oscillation, the period must pass through either a maximum or a minimum.

Condition for Minimum Period.

In the case of the pendulum already described, one portion of which is adjustable along a vertical line through the center of gravity of the other, it is not difficult to determine the condition for minimum period, in terms of the constants of the two parts, and their

relative position. It is sufficient to find the condition that the length of equivalent simple pendulum shall be a minimum, since the period will be a minimum at the same time.

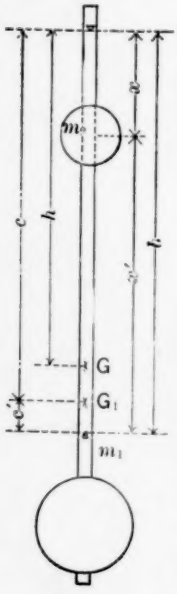

Fig. 1.

Let M be the mass of any pendulum, I its moment of inertia about the axis of suspension, and h the distance of the center of gravity from the axis. Then the length, l, of the equivalent simple pendulum is given by

$$l = \frac{I}{Mh}. \qquad (1)$$

In the pendulum represented in Fig. 1, the lower bob is rigidly attached to the rod carrying the knife-edge, these parts together constituting the "fixed" portion. The upper bob is the "movable" portion, and may be slid along the rod, and clamped at any point. G_1 represents the center of gravity of the "fixed" portion, while G is the variable center of gravity of the entire pendulum. Further, let

m_1 = mass of "fixed" part,
m_2 = mass of "movable" part,
$M = m_1 + m_2$ = total mass of pendulum,
I_1 = moment of inertia of "fixed" part,
I_2 = moment of inertia of "movable" part, } each about an axis through the *c. g.* of its own part, and parallel to the knife-edge,
c = distance (constant) of *c. g.* of "fixed" part from the axis,
x = distance of *c. g.* of "movable" part from the axis.

x and c will be taken positive when measured *downward* from the axis.

Then we have from equation (1), by the theorem of parallel axes, and by the definition of center of gravity,

$$l = \frac{I_1 + I_2 + m_1c^2 + m_2x^2}{m_1c + m_2x}, \qquad (2)$$

which gives the length of equivalent simple pendulum, in terms of

the distance of the movable mass from the axis. Equating to zero the differential coefficient of l with respect to x, we have, as the condition for a minimum (or maximum) value of l,

$$2x\,(m_1c + m_2x) - (m_1c^2 + m_2x^2 + I_1 + I_2) = 0,$$

which reduces to

$$m_2x^2 + 2\,m_1cx - (m_1c^2 + I_1 + I_2) = 0,$$

whence,

$$x = -\frac{m_1c}{m_2} \pm \frac{1}{m_2}\sqrt{m_1Mc^2 + m_2(I_1 + I_2)}. \qquad (3)$$

In the above quadratic in x, all the constants are essentially positive, while the constant term has a negative sign. Hence the roots given by equation (3) are always real and of opposite sign. Both values of x correspond to minima of l, and it is evident from equation (3) that these points are symmetrical about the point $-\frac{m_1}{m_2}c$, which is the position of the movable mass at which the value of l becomes infinite, since the moment about the axis here vanishes. The negative sense of the double sign in equation (3) therefore gives a large negative value of x, which brings the center of gravity of the pendulum above the axis; the pendulum then oscillating about the same axis in an inverted position. The positive sense of the double sign gives in all cases a positive value of x. Hence the following statement is perfectly general. *In any compound pendulum, if a portion of the mass having its center of gravity initially in the axis of rotation, be moved continuously downward, the period will pass through a minimum value.* In other words, the center of oscillation will first rise and then fall.

That this fact has not been clearly apprehended by writers on dynamics is evident from the following: Routh, Elementary Rigid Dynamics, 5th edition, Art. 93, Example 7. " * * * If a heavy particle be attached to a vibrating pendulum, it follows that the period is increased or decreased according as the point of attachment is at a greater or less distance from the axis of suspension than the center of oscillation." For adding at any point a particle

brought from outside the system is, as far as the effect on the period is concerned, exactly equivalent to sliding it to this point from either the axis or the center of oscillation.

Further, it may be shown that *the period reaches its minimum value when the distance of the center of gravity of the movable mass from the axis is one half the length of the equivalent simple pendulum.*[1] This may be seen by substituting in equation (2) the values of x given by equation (3). The former equation will then reduce to

$$l = 2x.$$

The length of equivalent simple pendulum, expressed as a function of the distance of the movable mass from the axis, is of the form (see equation (2))

$$l = \frac{m + nx^2}{p + qx},$$

in which m, n, p, q are real constants. To find the condition that a function of this form may have a maximum or minimum value for some positive value of x, we have

$$\frac{dl}{dx} = \frac{2nx(p + qx) - q(m + nx^2)}{(p + qx)^2} = 0,$$

or

$$x = -\frac{p}{q} \pm \sqrt{\frac{p^2}{q^2} + \frac{m}{n}}.$$

In order that x may have a real and positive value, it is necessary and sufficient that the constants shall be finite, and that m and n shall have the same sign. This condition is evidently fulfilled in equation (2). Hence, the period of a compound pendulum will *always* pass through a maximum or minimum as x increases from zero. This positive value of x corresponds to a *minimum* and not a *maximum* in the case of the compound pendulum. For the value of d^2l/dx^2 is found to be, after reduction,

$$\frac{d^2l}{dx^2} = 2\,\frac{np^3 + np^2qx + mpq^2 + mq^3x}{(p + qx)^4},$$

[1] For a special case see Love: Theoretical Mechanics, Art. 224, Example 3.

which is a positive quantity for positive values of x, in the case of the pendulum.

Application to the Reversible Pendulum.

If the pendulum is provided with a second knife-edge, vertically below the first, and at distance b from it (see Fig. 1), then if b be such that the second knife-edge is below the center of gravity of the system, the pendulum may oscillate in an inverted position, about the second knife-edge.

Condition for Minimum Period for the Inverted Pendulum. — By analogy with equation (2), we may write the length of simple pendulum equivalent to this "inverted" pendulum in the form

$$l' = \frac{I_1 + I_2 + m_1 c'^2 + m_2 x'^2}{m_1 c' + m_2 x'},$$

where c' and x' are measured downward from the axis. But since $c' = b - c$, and $x' = b - x$, we have, in terms of x,

$$l' = \frac{I_1 + I_2 + m_1 (b - c)^2 + m_2 (b - x)^2}{m_1 (b - c) + m_2 (b - x)}. \tag{4}$$

Equating to zero the differential coefficient of l' with respect to x, the condition for minimum period is found to be

$$x = \frac{Mb - m_1 c}{m_2} \pm \frac{1}{m_2} \sqrt{(Mb - m_1 c)^2 + m_2 (m_1 c^2 - Mb^2 + I_1 + I_2)}, \tag{5}$$

where, as in the case of the "erect" pendulum, the positive sense of the double sign is to be used in practical cases. The possibility of imaginary values of x may here be accounted for by the necessity of imposing a condition upon the constants in equation (4), in order to insure that the center of gravity of the system shall lie between the two knife-edges. Otherwise, the pendulum cannot, of course, be made reversible.

Condition for Reciprocal Axes. — Kater's reversible pendulum derives its usefulness from the fact that if the position of the movable mass be so chosen that the periods about the two knife-edges are equal, the distance between these knife-edges is then

equal to the length of the equivalent simple pendulum. The two axes of rotation are then said to be "reciprocal."

The condition that the axes shall be reciprocal is, therefore,

$$l = l' = b.$$

Substituting b for l' in equation (4), and solving for x, we obtain for the position of the movable bob,

$$x = \frac{b}{2} \pm \sqrt{\frac{b^2}{4} - \frac{1}{m_2}\{I_1 + I_2 - m_1 c(b-c)\}}, \qquad (6)$$

b being the distance between the two knife-edges. There are therefore *two positions* of the center of gravity of the movable bob which will give equal periods about the two axes, and these positions, when real, are *symmetrically situated with respect to the two knife-edges.* These may be termed the two "reciprocal positions" of the adjustable bob.

Experimental Verification. — In order to verify experimentally some of the preceding results, the constants of the Kater pendulum of this laboratory were measured with sufficient accuracy, the distance c being measured from the axis about which the pendulum swung when erect, *i. e.*, with the heavy end down. The constants as determined were

$m_1 = 3{,}915$ grams. $I_1 = 5.950 \times 10^6$ C.G.S. units.
$m_2 = 676$ grams. $I_2 = 0.008 \times 10^6$ " "
$c = 78.9$ cm. $b = 96.7$ cm.

On substituting these values in equation (3), we obtain

$$x = \begin{cases} 46.7 \text{ cm.} \\ -961.1 \text{ "} \end{cases} \quad \left\{ \begin{array}{l} \text{Positions of movable bob for} \\ \text{minimum period; pendu-} \\ \text{lum "erect."} \end{array} \right.$$

x being measured downward from the axis of suspension to the center of gravity of the movable bob. As previously noted, the negative value of x corresponds to an inverted position of the pendulum, which does not occur in practice.

Similarly, by substitution in equation (5), one finds

$$x = \begin{cases} 53.9 \text{ cm.} \\ 345.7 \text{ "} \end{cases} \qquad \left\{ \begin{array}{l} \text{Positions of movable bob for} \\ \text{minimum period; pendu-} \\ \text{lum "inverted."} \end{array} \right.$$

The periods corresponding to various positions of the movable bob were then observed for both "erect" and "inverted" pendulum, by coincidences with a standard pendulum, and these are exhibited in the accompanying curves. The computed minima agree well with the minima of the curves.

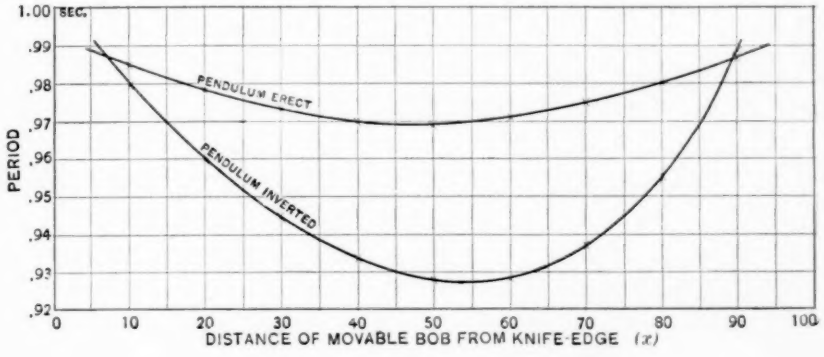

Fig. 2.

Further, substitution of the constants in equation (6) gives for the two positions of the movable bob at which the period is unchanged by reversal of the pendulum,

$$x = \begin{cases} 7.6 \text{ cm.} \\ 89.1 \text{ cm.} \end{cases} \qquad \left\{ \begin{array}{l} \text{Positions of movable bob for} \\ \text{reciprocal axes.} \\ \text{(Reciprocal positions.)} \end{array} \right.$$

At these two points the curve for the inverted pendulum and that for the erect pendulum should intersect; and the two points of intersection should have equal ordinates, since they correspond to equal periods. The curves are seen to conform sufficiently well to these requirements.

In the directions given by Stewart and Gee[1] for carrying out the adjustments of a reversible pendulum, it is stated that in order to obtain equality of periods, the adjustable bob is to be "always

[1] Practical Physics, Vol. I., p. 253.

moved toward that knife-edge about which the time of oscillation is the greater." It will be clear, however, from the foregoing considerations, that this statement is at best incomplete, and may be quite incorrect; because if it be true for the *upper* reciprocal position in any pendulum, it must of necessity be untrue for the *lower* one (as shown by the curves, Fig. 2); and if true in respect to an upper reciprocal position when this is *below* the axis, it must be untrue for this position when *above* the axis. Indeed, no instructions of this kind can be given, except in special cases where the reciprocal positions of the adjustable bob are approximately known beforehand.

General Statement of the Effect of a Movable Bob. — The behavior of a reversible pendulum provided with a movable bob may be summarized as follows: When the center of gravity of the movable bob is at its upper reciprocal position, the center of oscillation of the pendulum is at the lower knife-edge. If the movable bob be continuously lowered, the center of oscillation continuously rises, reaching a maximum height when its distance from the center of gravity of the movable bob is equal to the distance of the latter from the axis. When the center of gravity of the movable bob reaches a position as far below the mid-point between the knife-edges as it initially was above this point, the center of oscillation is again at the lower knife-edge. After this, it continues to fall indefinitely.

NORTHWESTERN UNIVERSITY, EVANSTON, ILL.,
October 1, 1903.

THE HEAT OF VAPORIZATION OF OXYGEN, NITROGEN AND AIR.

By J. S. Shearer.

In a recent determination of the heat of vaporization of air[1] it was observed that the results varied in a fairly definite way with the change of composition of the liquid. It seemed desirable to determine this constant for each of the constituents in order that a comparison could be made of the heat required to vaporize corresponding amounts of oxygen and nitrogen when separate and when mixed as liquid air.

The apparatus used was described in the former paper and the only changes found desirable were to use manganin wire instead of german silver, as the latter becomes very brittle after several immersions in liquid air, and to use a chronograph to record the times when meter readings were taken. The gases were liquefied by using liquid air as a cooling agent and subjecting the cold gas to considerable pressure. The apparatus used is shown in Fig. 1. Compressed gas from the cylinder C passed through an iron tube filled with caustic potash and then through a coil of fine copper tubing into a receptacle R. A gauge G indicates the drop in pressure due to condensation, and when the pressure became constant the coil and receptacle were known to be filled with liquid. On closing stopcock S_1 and opening S_2 the liquid could be removed without wasting gas from the cylinder. This operation was repeated

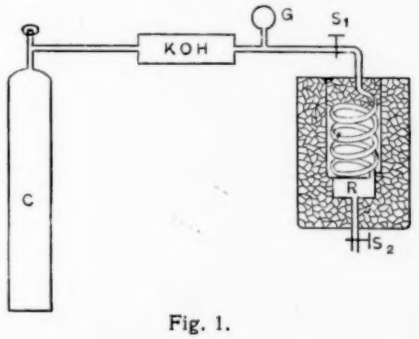

Fig. 1.

[1] Physical Review, Sept., 1902.

until the pressure in *C* became so low that further liquefaction was impossible. Oxygen liquefied very rapidly at pressures between 40 and 200 pounds per square inch but higher pressures are needed for nitrogen. A more economical method would be to use much greater pressures with small cylinders which could be entirely immersed in liquid air.

The oxygen was secured by electrolysis of water and after passing through caustic potash was found by analysis to be 98.8 to 99.9 per cent. pure. Atmospheric nitrogen was separated from oxygen by drawing air through iron tubes filled with copper turnings and heated above a red heat. The tubes used were two inches in diameter and about twenty inches long joined so as to make a rectangular coil giving approximately seventeen feet of copper packing. About 200 liters of nitrogen per hour was developed containing less than .5 per cent. oxygen. After filling the receiving reservoir, coal gas was passed through the hot coil, reducing the copper oxide, thus keeping up the activity of the copper.

The rate of vaporization was very constant when using the pure liquids even when considerable heat was developed by the coil, but when using liquid air there was a tendency to superheating and the meter readings were less regular. It is probable that the gas given off somewhat explosively when the current was used contained a larger quantity of nitrogen than that given off just before or just after this occurrence. Both of these facts tend to make it more difficult to get concordant results with liquid air than with the pure liquids.

A sample set of data for each gas may be of some interest.

Liquid Oxygen. December 19, 1902.

Duration of run,	828	sec.
Loss rate (no current) 1 liter in	76	sec.
Total amount of gas registered by meter,	45	liters.
Net amount due to energy from current,	34.1	liters.
Mean temperature of gas at meter,	16.3°	C.
Mean value of current,	1.22	amperes.
Mean potential difference at terminals of heating coil,	10.97	volts.
Barometric pressure,	74	cm. Hg.
Density of oxygen at 0° —76 cm. Hg,	1.429	gm. per liter.
Variation of voltage during run,	.02	volt.
Heat of vaporization,	59.5	(minimum).

Second run,	63.0	(maximum).
Mean of all six runs,	61.0	
Thermodynamic computation,	60.8	

The variation in the determinations of the temperature coefficient of vapor pressure is from 70 to 80, a much higher percentage than in the results of the heat of vaporization.

Nitrogen. March 27, 1903.

Duration of run,	309.6	sec.
Time to lose 1 liter (no current),	135.0	sec.
Total register by meter,	30.0	liters.
Net vaporization due to current,	27.71	liters.
Average current,	1.54	amperes.
Average voltage,	13.61	volts
Average temperature,	21.5°	C.
Barometer,	73.7	cm. Hg.
Density of nitrogen (grams per liter),	1.427	
Heat of vaporization,	49.73	calories per gram.

Another run required 308.2 seconds to vaporize 30 liters as against 309.6 above. The mean value found from a series of similar observations was 49.83; the greatest variation from the mean was .11. The result agrees very well with that computed from the vapor tension, which gives 49.25.

The composition of the vapor given off was found by pyrogallol absorption. The following convenient method was used to determine the corresponding liquid composition. The analysis of the liquid and vapor phases have been given by Baly and by Linde who allowed small portions of the liquid to vaporize completely in order to determine the liquid phase. If however readings of the meter are taken at small intervals from the start until *all* the liquid is vaporized and the vapor given off at each meter reading is analyzed a curve may be plotted using percentages of oxygen as ordinates and total amount of gas developed as abscissæ. The area of this curve gives the total oxygen content of the original liquid. The amount of oxygen in the liquid at any point is the area between the curve, the x axis, the ordinate at that point and the final ordinate. The composition of the liquid may thus be determined without in any way disturbing the process of vaporization. The results are given in the curves (Fig. 2). A comparison between the observed results and those computed assuming that the heat used for each component is

Per Cent. O.	Heat of Vaporization Observed.	Heat of Vaporization Computed.
48	50.6	54
66.5	57.9	56.8
90	59	59.8

TABLE I.

Curves I and II.

Per Cent. O Vapor.	Liters Evaporated.	Per Cent. O Liquid Computed.	Per Cent. O Vapor.	Liters Evaporated.	Per Cent. O Liquid Computed.
16.8	0	44.3	79	472	91.8
18.9	100	50	91.4	520	96.3
25	216	59	95.5	543	97.8
31.5	278	65.3	98	560.5	98.9
44.5	347	73.8		580.1 (final).	
61	419	83.5			

TABLE II.

Curves III and IV.

Per Cent. O Vapor.	Per Cent. O Liquid.	Per Cent. Total Gas.	Abs. T. [Baly].
16.8	44.3	0	80.7
17.5	47.3	10	81.0
19.5	51.0	20	81.3
22.2	55.1	30	81.7
26.1	61.5	40	82.4
33.3	66.6	50	83.5
44.6	73.8	60	85.1
58.0	81.5	70	86.7
77.0	91.0	80	88.8
91.9	96.2	90	90.2
		100	90.6

TABLE III.[1]

Curve IV.

T. Ab. T.C.	Per Cent. O Vapor.	T. Ab. T.C.	Per Cent. O Vapor.
80–193	12	86–187	52.3
81–192	17.6	87–186	60.3
82–191	24	88–185	69.5
83–190	30	89–184	79.2
84–189	36.8	90–183	89.6
85–188	44.4	91–182	

Boiling point of O 90.6°A. — 182.4°C.
" " " N 77.5° — 195.5°C.

[1] Baly, Phil. Mag., 49, 217. Also Traver's Study of Gases.

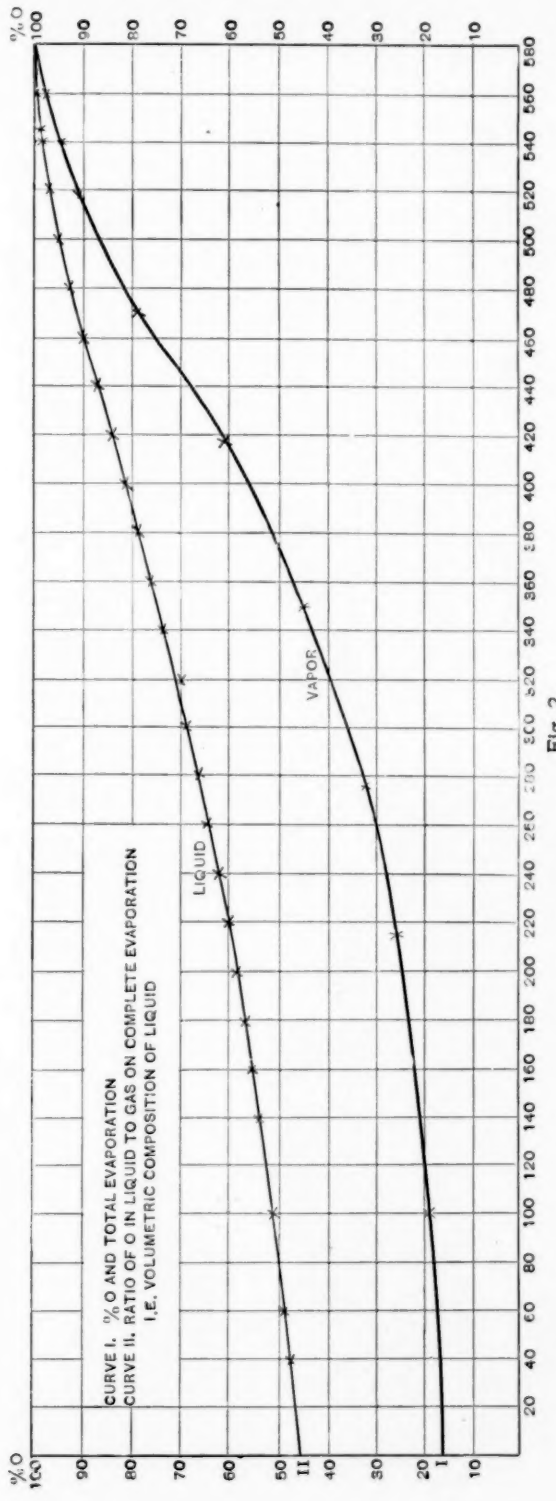
%O
100
90
80
70
60
50
40
30
20
10
CURVE I. % O AND TOTAL EVAPORATION
CURVE II. RATIO OF O IN LIQUID TO GAS ON COMPLETE EVAPORATION
I.E. VOLUMETRIC COMPOSITION OF LIQUID
LIQUID
VAPOR
II
I
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580

Fig. 2.

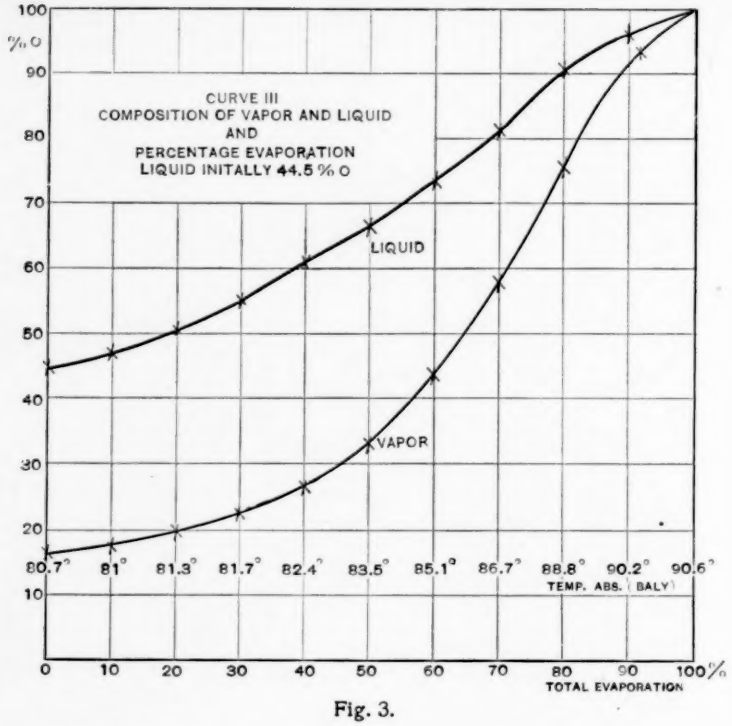
CURVE III
COMPOSITION OF VAPOR AND LIQUID
AND
PERCENTAGE EVAPORATION
LIQUID INITALLY 44.5 % O
LIQUID
VAPOR
% O
100
90
80
70
60
50
40
30
20
10
80.7°
81°
81.3°
81.7°
82.4°
83.5°
85.1°
86.7°
88.8°
90.2°
90.6°
TEMP. ABS. (BALY)
0
10
20
30
40
50
60
70
80
90
100 %
TOTAL EVAPORATION

Fig. 3.

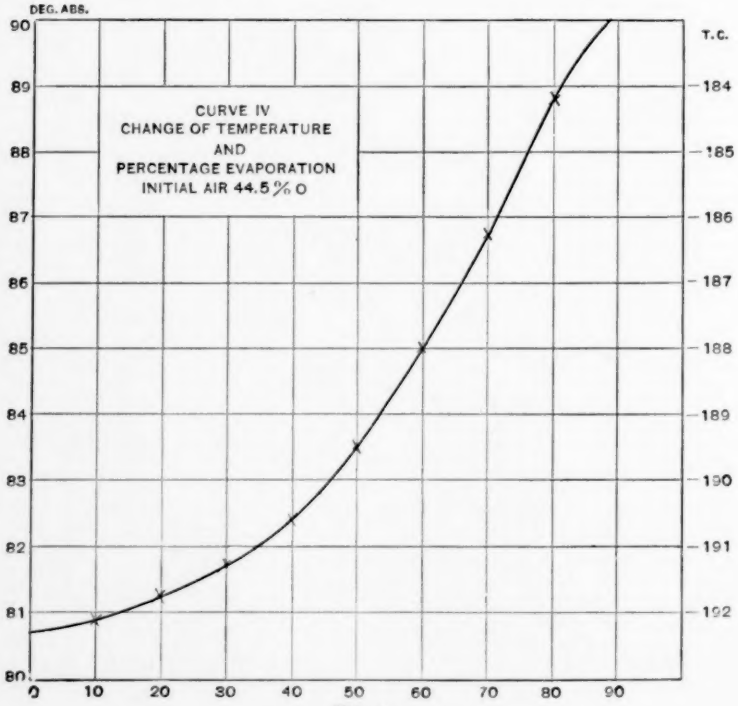
DEG. ABS.
T.C.
CURVE IV
CHANGE OF TEMPERATURE
AND
PERCENTAGE EVAPORATION
INITIAL AIR 44.5 % O
90
89
88
87
86
85
84
83
82
81
80
184
185
186
187
188
189
190
191
192
0
10
20
30
40
50
60
70
80
90

Fig. 4.

the same as would be required to vaporize the same amount from the pure liquids is shown below. The agreement is not very exact but is as close as could be expected owing to the difficulties noted above.

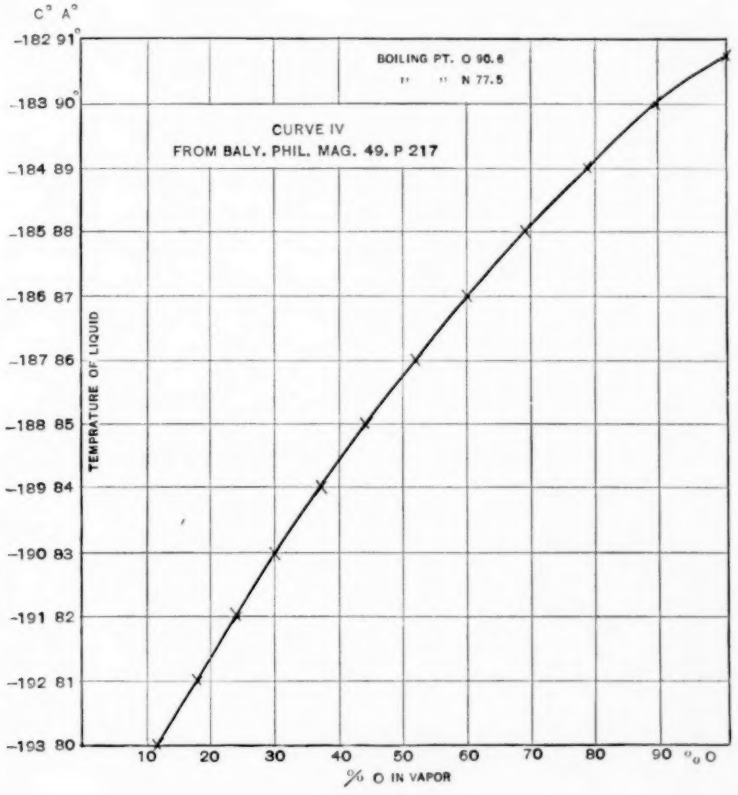

Fig. 5.

For convenience in reference curves are added showing the variation in temperature with percentage of entire liquid evaporated. The values of the temperatures for various compositions of the vapor phase were taken from Baly.[1]

PHYSICAL LABORATORY,
CORNELL UNIVERSITY, August, 1903.

[1] Phil. Mag. (49), p. 417, 1900.

THE SPECTRAL ENERGY CURVE OF A BLACK BODY AT ROOM TEMPERATURE.

By G. W. Stewart.

IN the past ten years much attention has been devoted to the study of the emission of solid bodies. An excellent résumé of the experimental and theoretical researches of Paschen, Lummer, Pringsheim, Wanner, Rubens, Kurlbaum, Wien, Rayleigh and Planck, is found in Vol. II. of Kayser's "Handbuch der Spectroscopie."

The measurements of the distribution of energy in the spectrum of a black body have been made with temperatures varying from about 1800° abs. to 373° abs. Langley[1] thought he obtained data for the distribution of energy in the spectrum of a black body at −20° C., and Mendenhall and Saunders[2] state that they found the energy curve of a black body at −90° C. It will be shown presently that the interpretations of these results were incorrect.

Lummer and Pringsheim[3] obtained a small portion of the spectrum of a screen at room temperature, used during their experiments.

The apparatus hitherto employed in the spectral energy curve measurements of a black body has consisted of a mirror spectrometer and a bolometer. The black body was placed in front of the slit of the spectrometer, the emitted energy passing through the spectrometer system and forming a prismatic spectrum at the bolometer strip slit. The change of temperature of the bolometer strip was then due to the resulting exchange of energy between it and the black body, and if there were enough difference between the room temperature and that of the black body, the energy emitted by the bolometer strip could be neglected. If one attempted to

[1] Am. Jour., of Science, Vol. XXXI., p. 1, 1886.
[2] Astrophys, Jour., 13, p. 25, 1901.
[3] Verh. d. D. Phys. Gesell., 2, 12, p. 163, 1900.

find the distribution of energy in the spectrum of a black body at the temperature of the room by this method, it would be necessary to make the temperature of the spectrometer and bolometer very much lower than that of the room.

This being quite impracticable, a different device was adopted in the work here described. A radiometer and mirror spectrometer were kept at the temperature of the room, and the temperature of the black body was lowered to that of liquid air. In such an arrangement, since the deflections of the radiometer are proportional to the resulting exchange of energy, and since the energy emitted by the black body is very small in comparison, the deflection obtained is really proportional to the emission of the radiometer vane to the black body.

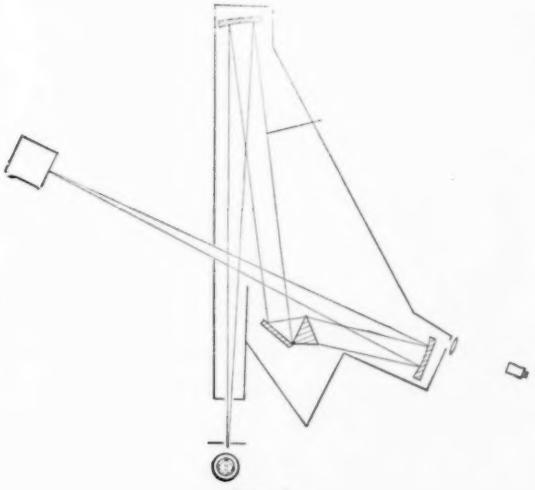

Fig. 1.

Langley and Mendenhall and Saunders failed to take this view of the matter. They seemed to think that their measurements gave them the distribution of energy in the spectrum of the black body, no matter what its temperature.

Apparatus and Adjustment. — The arrangement of the spectrometer and radiometer, Fig. 1, is practically the same as that described in a previous article.[1] The mirrors had a focal length of 55.7 cm. and

[1] PHYS. REV., Vol. XIII., Nov., p. 257, 1901.

an aperature of 9 cm. The mirror-prism device of Wadsworth [1] was used because the radiometer could be kept stationary, and because the radiation measured always passed through the prism at the angle of minimum deviation. The prism was of rock salt, and the refracting angle was 59° 47′ 53″.

Fig. 2 is a horizontal cross-section of the radiometer. The instrument is the same as that used in previous work, cited above, save a few modifications necessary for this particular experiment. *a* is the bronze block 5 cm. × 5 cm. with a vertical boring, *b*, 3 cm. in diameter; *a′* is a brass plate with a vertical slit soldered on the front of a horizontal boring *b′*, 2 cm. in diameter; *a″* is a brass tube with a vertical slit 2 mm. in width in its closed end, the tube being fitted in boring *b′*; *s′* and *s″* are rock-salt plates; and in the center of *b* and in front of *s″* are the vanes. Plate *s′* was used in order to obtain a vacuum, while *s″* serves as a wall which is essential for a sensitive radiometric action.

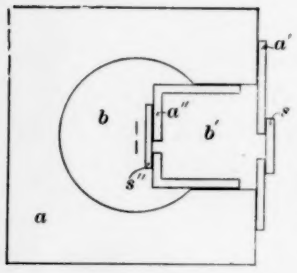

Fig. 2.

The radiometer vanes were constructed to approach a perfectly black body as nearly as possible. Although the radiating vane is situated in a hole, yet its deflection depends not upon the total amount of energy radiated from the hole, but upon only the energy emitted by the vane. The vanes were made of platinum and coated with platinum-black. Kurlbaum [2] has shown that a coating of platinum-black, 200 gr./dm.² at 100° C. emits about 97 per cent. of the radiation of an ideally black body at that temperature. In order to assure a thickness of at least 200 gr./dm.², about four times this amount was deposited electrolytically according to Kurlbaum's method. While the vane emitted at least 97 per cent. as much energy as an ideally black body at 100° C., it does not follow that the distribution of energy in the spectrum of the radiometer vane at room temperature was approximately the same as that of an ideally black body at the same temperature. This point will be discussed in another part of this paper.

[1] Phil. Mag., 38, p. 337.
[2] Annal., 67, p. 846, 1899.

The rotation of the radiometer vane was read by means of a telescope and scale.

Fig. 3 shows the apparatus which was placed in front of the spectrometer slit. The black body was an oblong tin box, with top, heavily coated on the interior with lampblack. A V-shaped ridge was placed in the bottom in order to diminish direct reflection. This approximately black body was suspended in liquid air in a Dewar bulb, the latter being raised or lowered until the box just floated. The black body was thus kept at the same position. Above the bulb was placed a shutter made of glass and tin, and above this a concave mirror of about 11 cm. radius and 7 cm. aperture. The mirror, shutter and bulb were placed in front of the spectrometer slit, the distances being such that the image of the slit of the black body fell upon the spectrometer slit. While this "black body" was not ideally black, yet, as will be shown, it was nearly enough so.

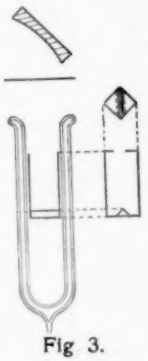

Fig 3.

The prism table was first so adjusted that sodium light from the spectrometer slit passed through the prism at the angle of minimum deviation. The second mirror was then rotated so that the image of the spectrometer slit fell upon the radiometer slit, and the radiometer was at a known wave-length, that of the D-lines.

For observations in different parts of the spectrum, the prism table was rotated through an angle which was one half the change in minimum deviation.

Light striking the radiometer slit would be partly reflected from the rock-salt plate which was just behind. The lens shown near the second mirror in Fig. 1 was so adjusted that this reflected light fell upon it and was brought to a focus in the microscopic eye piece. By looking in the eye piece, the observer could determine when the image of the spectrometer coincided with the radiometer slit. This adjustment could be made by rotating the prism table to within 10″ as read on the spectrometer, and 10″ error would mean less than .01μ at any wave-length here considered.

The relative position of the vane and the radiometer slit was quite important, for the vane and slit were of the same width and if the

vane were not directly behind the slit, the effective slit would be not only narrower, but displaced. This displacement would produce an error in the computed wave-length. In order to avoid this error, the vane was adjusted as correctly as possible with the eye, and then the apparatus was tested by finding the position of the CO_2 emission band in an acetylene bunsen burner flame. If the maximum did not come at 4.40μ which is given by Paschen,[1] a leveling screw was turned until the vane was properly adjusted.

RESULTS.

The distribution of energy in the spectrum of the approximately black body, the radiometer vane, was obtained by reading the deflections with the shutter open and closed. A number of observa-

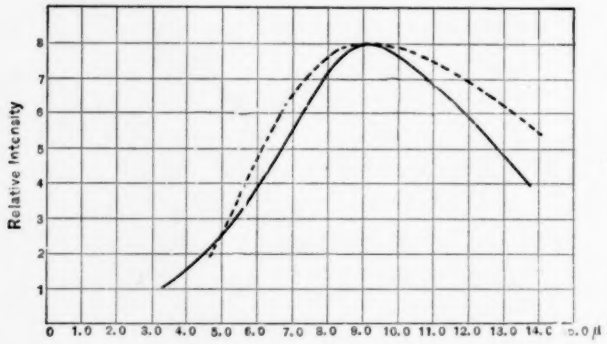

Fig. 4.

tions were taken for each setting of the prism table, and the average retained. These are the observed points in Fig. 4. The observed bands at 4μ to 5μ and 6.2μ, shown by a dotted line, are those of CO_2 and H_2O respectively, the absorption being due to the gases occurring in the atmosphere of the room.

Paschen[2] states that the absorption bands of the CO_2 occur at about 4.27μ, and that there are a number of absorption bands of H_2O in the neighborhood of 6μ, giving a maximum at perhaps 5.95μ.

The wave-lengths for the curve were computed from the indices or refraction of rock salt given by Rubens and Trowbridge.[3] In the

[1] Annal., 53, p. 334, 1894.
[2] Annal., 53, pp. 334-336, 1894.
[3] Annal., 61, p. 724, 1897.

accompanying Fig. 5, the lower curve is plotted with indices and wave-lengths as coördinates, and the upper curve gives the relation between the rotation of the prism table from the *D*-lines and the computed wave-lengths.

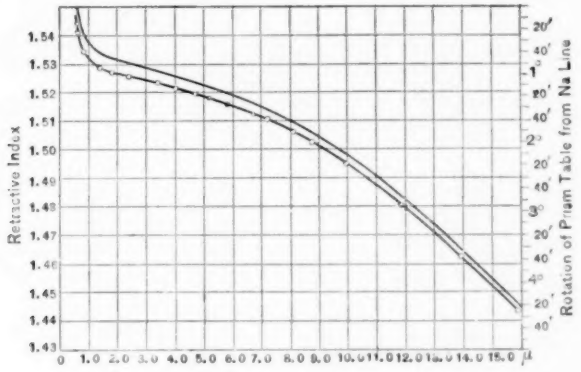

Fig. 5.

In Fig 6, the full line is the actual distribution of energy, the correction for slit width being made according to Paschen's[1] method. The maximum intensity occurs at 9.2μ. It is of interest to compare this maximum with that which would be predicted by the equation $\lambda_m T = A$, λ_m being the wave-length of the maximum, T the

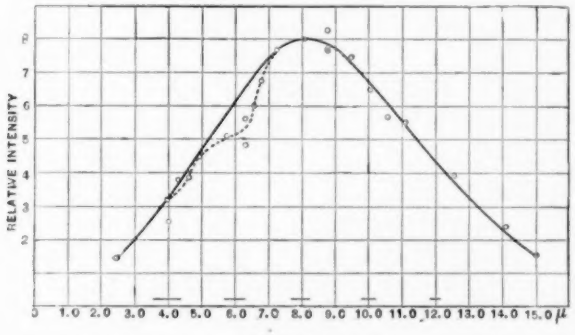

Fig. 6.

absolute temperature and A a constant, determined experimentally at other temperatures. The most reliable determinations of A thus far made are those of Paschen and of Lummer and Pringsheim.

[1] Annal., 60, p. 712, 1897.

Paschen's [1] value is 2920, and Lummer and Pringsheim [2] have found 2940. The temperature of the room during the experiments here described was about 24° C., or 297° abs. Using the latter value for T and Paschen's value of A, the wave-length of the maximum becomes 9.8μ, which does not agree with that observed, 9.2μ.

There are several sources of error in the experimental work but, as will be seen, they do not seem to account for the difference between the predicted and observed position of the maximum. Neither the radiometer vane nor the box at liquid air temperature is an ideally black body. The latter does not need to be even an approximately black body for the maximum of its spectrum is at such a great distance from 9μ that its relative absorption in this region would be practically the same as that of a black body. As stated before, the vane was covered with platinum-black to such a thickness that it would at 100° C. emit more than 97 per cent. of the radiation of an ideally black body. The wave-length of the maximum emission at that temperature would be 7.8μ, and hence the energy in the neighborhood of 9μ and 10μ would be a considerable portion of the entire energy. If the difference between the total emission of the vane and that of a black body, both at 100° C., is less than 3 per cent., and if this 3 per cent. error is distributed, and not in one region, then it seems quite impossible for the maximum of the emission of the vane at 24° C. to differ appreciably from the maximum of an ideally black body.

It is interesting to compare the results of these observations with a curve obtained from the equation of Planck. His equation is

$$J = c_1 \lambda^{-5} (e^{\frac{c_2}{\lambda t}} - 1)^{-1}.$$

Here J is the energy at any wave-length λ, T is the absolute temperature, and c_1 and c_2 are constants. The value c_2 is, from theory, $4.965 \times \lambda_m T$. If, for the sake of comparison, the value for $\lambda_m T$ found in this experiment be used, c_2 becomes 13,530.

Using this constant, the relative values of J/c_1 are plotted by a dotted curve in Fig. 6. As will be seen, the theoretical and experimental curves do not agree.

[1] Annal., 4, p. 277, 1901.

[2] Verh. d. D. Phys. Gesell., 1, p. 215, 1899.

While I am at a loss to understand why my results do not agree with those demanded by a well-verified theory, yet I do not urge them as any objection to the theory. The maximum deflection obtained was only about 4 mm. on a scale 64 cm. from the mirror, but the consistency of the results annuls any criticism on the basis of the minuteness of the deflections.

The experimental work was performed at the Physical Laboratory of Cornell University.

STATE UNIVERSITY OF NORTH DAKOTA,
September, 1903.

A CONVENIENT GALVANOMETER.

BY WALTER P. WHITE.

THIS instrument is the result of an attempt to design a galvanometer for elementary work. The requirements set were: that it should be reliable, accurate, quick-acting, direct reading, portable, and capable of being constructed in the humblest high school laboratory. The result seems to justify the expectation that the instrument may prove useful in advanced, as well as elementary, work. Comparing it with the Weston milammeters, it is at least as accurate, can be read as quickly and easily, and has the same resistance for a given sensibility. It has the disadvantage that its indifference to outside magnetic disturbances, though great, is not complete — on the other hand, it costs far less than a good commercial ammeter, and has other less fundamental advantages.

No one feature of the galvanometer is new — indeed, the combination is, in a general way, such as follows naturally from an application of the reflecting principle in a field where an adherence to the old-fashioned tangent galvanometer type with needle and pointer has effectually prevented the development of satisfactory instruments. The reflecting galvanometer, on account of the smaller deflection used and the greater radius of the scale, does not require so careful centering of the needle, and hence a simple instrument is in this respect as good as a very expensive galvanometer with a pointer. The short needle is more quick acting. The shortness of the needle and the smaller deflection combine to give a small displacement of the poles of the needle; the diameter of coil necessary to make the instrument obey the tangent law is thus very much reduced, and from this fact follow several advantages. First, it becomes convenient to use a movable magnet system of two needles, with poles oppositely directed, such as are universally employed in sensitive astatic galvanometers. The fact seems to be often overlooked that

this arrangement is valuable for securing freedom from magnetic disturbances in less sensitive instruments also. The use of this kind of a magnet system, which by itself would be astatic, requires, of course, the addition of a control magnet. Second, the small coil renders it possible to make the resistance several hundred times as small, for the same sensibility. In practice, it is better to sacrifice some of this gain and make the coil more powerful, correspondingly increasing the magnetic control. In this way we keep the needle under the action of comparatively strong forces, and thus obtain a freedom from the action of convection air currents inside the case which is quite as important as freedom from magnetic disturbances.

Freedom from both magnetic and mechanical disturbances is thus promoted by the use of a control magnet. In addition, such a magnet renders it very easy to regulate the sensibility of the galvanometer so that the reading is given directly in some accepted unit. A third advantage is that constancy which can be secured in a modern laboratory only by being independent of what is by courtesy called the earth's field. The ability, also, to use the instrument in any desired position regardless of the points of the compass is often a decided advantage. This advantage is limited, however, by the fact that the absolute sensitiveness varies by a fraction of one per cent. in different orientations.

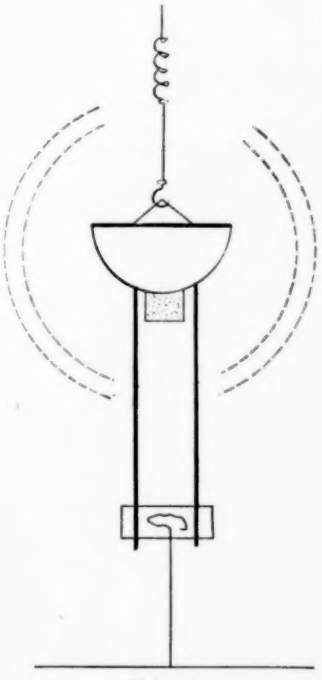

Fig. 1.

For the magnet system the Weiss form has been selected, in which there are two vertical needles, each end of the combination having two unlike poles, belonging to different magnets, but acting like the two poles of a horizontal magnet whose length is equal to the distance between the two needles. Such a system has practically permanent astaticism,

and is far easier to construct than the usual form, especially for a novice. The needles actually used are 1.2 mm. thick and 43 mm. long, placed 8 mm. apart (see Fig. 1). The weight is about a gram, which gives stability. The astaticism is so good that, with the control magnet producing a field equivalent to about eight times that of the earth exerted on each pole, the maximum disturbing effect of the actual earth's field usually causes a deviation of about fourteen minutes of arc. Better results might be obtained, for the assembling of each system took only two or three minutes, and consisted in laying the needles, as nearly parallel as could be judged by the eye, on two strips of mica, and fastening them there with sealing wax, applied with a warm nail. It would require, however, considerable skill and more patience to approach the above result by building up a system of the ordinary type.

The damper is a horizontal wire, 4 cm. long, turning in a shallow layer of clean kerosene. Air and magnetic dampers are out of the question for portable galvanometers with fiber suspension, on account of the very small clearance required if they are to be effective. The damper here described, however, is more than a mere substitute — it is in many respects preferable even for stationary instruments. It is easier to make, never requires adjustment, is not disturbed by dust or displacements of the galvanometer, and suffers far less from convection currents than an air damper.

That the old fashioned type of tangent galvanometer has often been preferred to the cheaper, more accurate, and more efficient mirror galvanometer is, perhaps, mainly due to the following reasons: The slowness of the reflecting galvanometer when damped in the usual way; its greater sensitiveness to disturbances (when used in the "earth's field"), and the trouble that fiber suspensions give by breaking. It has just been shown that the two first considerations are altogether in favor of the mirror instrument, when it is properly made; the third difficulty has been avoided by hanging the suspension from a small spring, which weakens the effect of shocks, and by using a bundle of eight silk fibers instead of one. The torsion of such a bundle, 1 cm. long, produces an altogether negligible error. (It may be worth while to call attention to the fact that it is not the amount of the torsion, but the extent to which

its effect varies from that of an approximately equal amount of magnetic control, which determines the error in an empirically calibrated instrument.)

In order to make the galvanometer direct reading, the scale is bent to a curve such that distances measured along it are proportional to the tangent of half the angle subtended. When a template has once been made it is as easy to make such scales as to make straight ones. A board can be sawed in two along the proper curve and then screwed together again, clamping one edge of a flexible scale in the cut. An error of as much as a a millimeter in the saw cut causes an error in the scale readings which is inappreciable, even when the scale distance is only 25 cm. To obtain the proper curve, a circular arc, *MAN*, Fig. 2, is described with radius, *OA*, equal to the proposed scale distance; radii are drawn two degrees apart; these radii are extended beyond the circumference as follows:

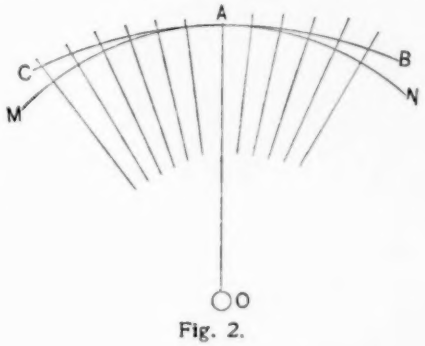

Fig. 2.

At	2	4	6	8	10	12	14	degrees from *A*
Add	.0002	.0008	.0021	.0036	.0055	.0085	.0108	of the radius.
At	16	18	20	22	24	28		degrees from *A*
Add	.0143	.0183	.0227	.0274	.0327	.0440		of the radius.

A smooth curve, *CAB*, is drawn through the points thus located. The readings will be proportional to current with uniform scale divisions of any length. The zero must be at *A*, and the scale distance equal to *OA*.

These numbers were determined by triangulation, using six-place tables. Three different systems of triangulation gave concordant results.

The scale is read by the double mirror system. A small stationary mirror, either below and behind or above and in front of the

swinging one, gives an image of a line somewhere on the scale. If the mirror nearest the eye is thin, this line appears to come close up to the scale as seen in the other mirror, and thus serves as an index. Since the eye is in focus for both images at the same time, this arrangement dispenses with a telescope. At 25 cm. scale distance, divisions 2 mm. long can be read to tenths. A telescope is decidedly preferable, however, for those who have learned to use it without undue effort. With a telescope 13 cm. long one can with much greater ease read millimeter divisions, thus doubling the range of the instrument.

There are two coils, of the Helmholtz type, each 4 cm. in diameter. The ratio of magnet length to coil diameter is thus 1 : 5. With this ratio, and with each half of the scale subtending an angle of 28°, the greatest possible error due to irregularity of the field is about 1/2,000 of the length of the scale.[1] Fig. 1 shows the magnet system with suspending spring and fibers, hemispherical mirror, and damping wire, full size. The upper piece of mica is behind the mirror. The dotted circles show the size of the coils, and the small dotted square shows the position of the stationary mirror. For a sensibility of one milliampere there may well be twenty turns in the coils, which should have a resistance not over .1 ohm.

The writer has used one of these instruments for a great variety of work. Its coils have 160 turns, 3 ohms resistance, and the reading is in tenths of milliamperes, with shunts for milli- and centiamperes. A calibration showed no error greater than .001 the length of the scale. The instrument is not necessarily quite as accurate as this, since there may have been errors in reading, or in the standard resistances, greater than any observed — still, an in-

[1] Half this error is that inseparable from a perfect Helmholtz galvanometer, and the other half is the error which would result if the true distance of the coils were one millimeter in error. (There is an even chance that the two errors may oppose each other.) The unavoidable error varies approximately as the fourth power of the ratio of magnet length to coil diameter, the other, as the square of this ratio, and also as the square of the ratio of the error in distance to the coil diameter. Hence if the diameter of the coil is increased the gain in accuracy will be as the fourth power of the ratio of increase, but if the magnet is made shorter the gain will be something more than the square. "Error" here means, as is appropriate for an empirically calibrated instrument, the greatest discrepancy between readings, which is far less than the variation from the reading of a very short magnet, the quantity which is given in the usual formulas.

strument whose inaccuracy can not be detected through the use of a good resistance box is good enough for many kinds of work. Even more gratifying than the accuracy, however, are the quickness and the certainty of the readings, which, in comparison with the generality of even excellent galvanometers, are very striking.

Most of the features above described are already familiar in other instruments, or are such that their advantages can be readily estimated. A fluid damper, however, seems often to be considered liable to cause large errors through surface tension effects.

Otherwise, such dampers would prove valuable — they are practically indispensable in a portable reflecting galvanometer and would greatly improve many existing instruments, to which they could be easily applied. To test the magnitude of the surface tension effects, a magnet system was observed in which they were made 2,500 times as great as need be, by making the damper attachment very eccentric, and by moving the wall of the containing vessel to and fro through a space of 8 mm., which gives 64 times the variation of distance that would occur with a damper centered to within .5 mm. and turning through an angle of 14°. Not the slightest effect was observable in a telescope half a meter away. The magnet system weighed 40 gm., and its time of vibration was .25 second. Increasing the time to 10 seconds would make the system 1,600 times as sensitive to disturbances, but the substitution of a well-made damper would evidently more than offset this. We may say, then, that the surface effect may be made negligible for any magnet which is heavy enough to need such damping at all. The fluid in this case was ordinary kerosene, and the diameter of the containing vessel, 7 cm. The necessary diameter can readily be found by trial in any particular case. A horizontal wire is better than a vane, in that it allows the use of a very shallow layer of fluid. Convection currents are much smaller in such a layer, and waves, if any occur, are more quickly damped out.

CORNELL UNIVERSITY.

NOTE.

BY J. E. TREVOR.

Josiah Willard Gibbs.—In the death of Professor Josiah Willard Gibbs, in April 1903, there passed away a man whose work has left a lasting impression on the course of the development of physical science. Gibbs's life was the quiet and outwardly uneventful one of a scholar. The son of Josiah Willard Gibbs, the widely known philologist and Professor of Sacred Literature in the Yale Divinity School from 1824 to 1861, he was the sixth of the family bearing this name. His schooling was obtained in New Haven; and he was graduated from Yale College, where he won a number of undergraduate distinctions in Latin and Mathematics, with the class of 1858. After five years of graduate study at Yale he obtained the degree of Doctor of Philosophy, and then became a tutor in the college. In this capacity he taught Latin for two years and "Natural Philosophy" for one, and then went abroad for further study in physics and mathematics. The winter of 1866–67 he spent in Paris, the following winter and spring in Berlin, where Magnus then was, the third winter in Heidelberg under the influence of Kirchhoff and Helmholtz, and the succeeding spring again in France. Gibbs then returned home, and in 1871 was appointed Professor of Mathematical Physics in Yale College, which position he filled for the remainder of his life.

Gibbs's published researches in physics were in the fields of thermodynamics, the electromagnetic theory of light, and statistical mechanics; his work in pure mathematics was in vector analysis and multiple algebra. Of them all, his thermodynamic studies have perhaps brought him the most fame. In the first of these, Graphical Methods in the Thermodynamics of Fluids,[1] he examines the advantages of using various pairs of thermodynamic variables for

[1] Transactions of the Connecticut Academy, 2, 309, 1873.

graphical representation. The ordinary pressure-volume diagram, in which areas represent quantities of work, the temperature-entropy diagram, in which they represent quantities of heat, and the volume-entropy diagram, among others, are discussed in a most interesting way. This study led naturally to the subject of the next paper, A Method of Geometrical Representation of the Thermodynamic Properties of Substances by means of Surfaces.[1] A thermodynamic surface, in which the coördinates are pressure, volume, and temperature, had been suggested by James Thomson; but Gibbs here describes a far more general one, the surface

$$\varepsilon = f(v, \eta),$$

in which ε is the energy, v the volume, and η the entropy of a body. This surface furnishes the only geometrical representation of *all* the stable and instable states of a body of constant composition: every other fails in some respect with the appearance of two-phase or three-phase states. Because of the differential equation

$$d\varepsilon = -pdv + td\eta,$$

the position of the tangent plane at any point of the surface represents the pressure and temperature corresponding to the point. After establishing in most ingenious fashion the criterion of equilibrium under imposed conditions of constant pressure and temperature, Gibbs is enabled to indicate the form of the analytical surface $\varepsilon = f(v, \eta)$ throughout the entire range of stable and instable states; and thereupon to construct the "derived surfaces" that represent the possible multiphase states. The whole supplies a remarkably beautiful representation of the theory of the analytical continuity of the primitive one-phase state. This paper attracted Maxwell's attention to such a degree that he devoted a number of pages of his well-known Theory of Heat to an account of the surface; and he constructed a model of it, a cast of which was presented to Gibbs. A very interesting analytical exposition of the properties of Gibbs's energy surface has been recently given in a paper by Paul Saurel.[2]

[1] Trans. Conn. Acad. 2, 382, 1873.

[2] Jour. Phys. Chem. 6, 474, 1902.

These studies in the graphics of thermodynamics led naturally to the results of the epoch-making paper, On the Equilibrium of Heterogeneous Substances,[1] in which the criteria of thermodynamic equilibrium are applied to systems of bodies differing in chemical nature and even subject to reversible chemical interaction. This investigation is one of the most imposing results of the scientific activity of the nineteenth century, and is widely considered to be Gibbs's most important contribution to physical science. Its extent is tremendous; although compact to a degree, it occupies over three hundred closely printed pages, and contains seven hundred numbered equations.

Gibbs begins by showing that the fundamental criterion of thermodynamic equilibrium, that the entropy of a material system must be a maximum under imposed conditions of constant energy and volume, can be transformed into the analytically more serviceable theorem that equilibrium will subsist when the energy is a minimum under the conditions of constant volume and entropy. He then regards the energy of a homogeneous mass of variable composition as a function of the volume v of the mass, the entropy η, and the masses m_i of the independently variable components, thus establishing the equation

$$d\varepsilon = -pdv + td\eta + \mu_1 dm_1 + \mu_2 dm_2 + \cdots + \mu_n dm_n,$$

in which p, t are the pressure and absolute temperature of the mass, and the quantities μ_i, termed the "potentials" of the components, are defined as the mass derivatives of the energy. Influences due to gravity, distortion of the solid masses, capillary tensions, and electricity are temporarily neglected. Application of the criterion of equilibrium to a system of coexistent phases then yields the important general result that, for equilibrium, each potential must have the same value in all the phases.

Integration of the above differential equation under the supposition that the mass of the phase increases from zero to a finite value, its nature and state remaining unchanged, yields

$$\varepsilon - pv + t\eta = \mu_1 m_1 + \mu_2 m_2 + \cdots + \mu_n m_n,$$

[1] Trans. Conn. Acad. 3, 108, 1896; 3, 343, 1878.

a relation which asserts that the thermodynamic potential of the phase is a linear function of the potentials of the components. It is then shown that, because of the form of the differential equation for ε, all the $n + 3$ independent relations between the $2n + 5$ variables involved can be found by differentiation when the form of the function $\varepsilon(v, \eta, m_1, m_2, \cdots, m_n)$ is known. By analytical transformation of the differential equation, it is shown that the same result follows when the form of the free energy $\psi(v, t, m_1, m_2, \cdots, m_n)$- of the heat-function $\chi(p, \eta, m_1, m_2, \cdots, m_n)$, of the thermody, namic potential $\zeta(p, t, m_1, m_2, \cdots, m_n)$, or of a function $f(p, t, \mu_1, \mu_2, \cdots, \mu_n)$, is known. Gibbs terms the equations $\varepsilon = \varphi(v, \eta, m_1, m_2, \cdots, m_n)$, etc., "fundamental equations." This important demonstration, that the study of the thermodynamic properties of a phase is completed when the form of one of its fundamental equations is determined from experiment, is a generalization and extension of Massieu's theory of the "characteristic functions" of fluids.

Application of the equations to a system of r coexistent phases leads now to the famous "phase rule" — the conclusion that such a system is capable of but $n + 2 - r$ independent variations of state. Elimination of differentials from the r phase-equations

$$0 = -v dp + \eta dt + m_1 d\mu_1 + m_2 d\mu_2 + \cdots + m_n d\mu_n$$

then supplies for the ratios of the differentials dp, dt, $d\mu_1$, $\cdots$ expressions in terms of the volumes, entropies, and compositions of the several phases. In the case of the derivative dp/dt, these results constitute a most interesting generalization of the Clapeyron-Clausius equation.

Referring back to the paper on the energy surface for a one-component system, it is pointed out that such a geometrical representation is furnished by each of the surfaces

$$\varepsilon = f_1(v, \eta, m) \qquad \chi = f_3(p, \eta, m)$$
$$\psi = f_2(v, t, m) \qquad \zeta = f_4(p, t, m),$$

these surfaces bearing definite relationships to one another, and the most comprehensive of them being the first. Gibbs then shows how the equation

$$\zeta = f(p, t, m_1, m_2, \cdots, m_n)$$

can be employed for a representation, by means of families of surfaces, of the equilibria of two-component and three-component phase systems, an intricate matter which he develops in considerable suggestive detail. In all such applications of the theory Gibbs's introduction of fundamental equations has been very clarifying, giving unity to the subject and making clear what relations must be found from experiment to the end of making possible the complete integral treatment of thermodynamic problems by purely analytical means — in his phase, "giving shape to research."

The remainder of the first half of the paper, the part published in 1876, contains two applications of great interest. In a discussion of the values of the potentials when the quantity of one of the components is very small, Gibbs distinctly outlines the general features of the celebrated theory of dilute solutions formulated by van't Hoff ten years later, carrying the matter as far as was permitted by the experimental facts available at the time. The second application is the theory of dissociation of gases. The condition of dissociation-equilibrium is that the potentials of the proximate components of the gas mixture shall satisfy an equation of the same form as that obtaining between the units of weight of these components. This theory of dissociation was tested by assuming the components to obey the laws of ideal gases, and from the fundamental equations of such gases calculating the forms of the potentials of the components. The result was cast into the form of a relation between the density and the pressure and temperature of the mixture, and was found to exhibit a striking agreement with the existing determinations, made by Deville and Troost and by Playfair and Wanklyn, of the densities of nitrogen peroxide. This test was extended[1] in 1879 to a calculation of the "abnormal vapor densities" of formic and acetic acids and of phosphorus perchloride.

The second half of the paper, issued two years after the appearance of the first, commences with a deduction, from the general criterion of equilibrium, of the conditions of equilibrium for solids in equilibrium with fluids with regard to all possible states of strain of the solids. This is followed by a thermodynamic theory of capillarity, and by a study of the electromotive forces of reversible cells,

[1] Am. Jour. Sci. (3), 18, 277, 1879.

the distinguishing feature of which is the inclusion of the term $td\eta$ in the expression for the differential of the energy of the cell. A most interesting abstract of the whole paper, by its author, appeared in the sixteenth volume of the American Journal of Science.

The method of this paper, being an analytical development of the consequences of inductively established general postulates, is of great effectiveness, and it is applied with remarkable physical insight and mathematical skill. The significance and importance of the results attained were not at first realized; partly no doubt because of what has been called the "logical austerity" of the method, but certainly in considerable measure because the work appeared in a publication not generally accessible to scientific workers. Because the work remained for a long time virtually unknown, some of its results were rediscovered by others in later years. A striking instance of this was Helmholtz's celebrated development, in 1882, of the theory of free energy; which was essentially a development of the properties of Gibbs's function ψ.

Yet, now, a quarter of a century after their publication, Gibbs's results are powerfully influencing experimentalists and theoreticians alike. This is especially true of the phase rule, and of the theory of the critical states of systems of two or more independently variable components. The phase rule, to which attention was first directed by the Amsterdam physicist van der Waals, in the hands of Roozeboom and his followers, Schreinemakers, Bancroft, and others, has brought order into an apparently inextricable tangle of chemical and physical facts. One of the most extended single applications of it is van't Hoff's recent study of the very complex equilibria subsisting between the solid phases deposited in the evaporation of sea water; and the "rule" has of late shed much needed light on the operations of the manufacture of steel. Every recent treatise on thermodynamics bears evidence of the deeply modifying influence that Gibbs's work has exerted on the subject. The most extensive of these, Duhem's monumental work on "chemical mechanics," is essentially an exposition and development of Gibbs's theory.

A distinct service was rendered to contemporary physics and chemistry by Ostwald's publication, in 1892, of a German version[1]

[1] Thermodynamische Studien, Leipzig, 1892.

of the three papers from the Transactions of the Connecticut Academy. This was followed, in 1899, by Le Chatelier's translation[1] of the first part of the "Equilibrium of Heterogeneous Substances." This paper in particular, as a scientific classic, must eventually be made generally accessible in English: we cannot indefinitely content ourselves with the not very readable German text and the incomplete French one. What is badly needed is a collected edition of all Gibbs's thermodynamic publications.

After the publication of these thermodynamic researches, Gibbs's creative scientific activity lay in the fields of multiple algebra and vector analysis, and in the electromagnetic theory of light and statistical mechanics. Feeling, with other physicists, the need of a simple and direct algebra of vectors, he developed such an analysis in which the product of two vectors is so formed as to give a positive value. This avoided the loss of naturalness in the Hamiltonian system of quaternions, due to the fact that it causes the square of a vector to become negative. A privately circulated outline of this vector analysis, issued in part in 1881 and completed in 1884, attracted considerable favorable attention, which interest has been transferred to the authorized systematic exposition of the theory prepared by Dr. E. B. Wilson two years ago. In the intervening years Gibbs developed a number of applications of the new analysis to problems of astronomy and physics, and made original contributions to the broader subject of multiple algebra in general.

Between 1882 and 1889, Gibbs published a series of papers on the electromagnetic theory of light, and its relations to the various elastic theories. By simple and direct methods, and with a notable absence of special hypotheses regarding a connection between matter and ether, he gave the first adequate explanation, on the basis of the theory, of the dispersion of colors, and developed a number of consequences concerning refraction and polarization. Of his arguments for excluding the theories of light other than the electrical, it has been said that "they would have sufficed to firmly establish this theory even if the experimental discoveries of Hertz had not rendered such discussions forever unnecessary."[2]

[1] Equilibre des Systèmes chimiques, Paris, 1899.

[2] Henry A. Bumstead, Am. Jour. Sci. (4), 16, 197, 1903.

In his Elementary Principles in Statistical Mechanics, published in 1902 as one of the Yale Bicentennial Publications, Gibbs returns to an aspect of the subject of his first researches — thermodynamics. In the endeavor to deduce the laws of thermodynamics from the principles of mechanics, Clausius, Maxwell, and Boltzmann, among others, were led to study the approximate and probable behavior of systems of so great a number of particles that we have not the fineness of perception to enable us to appreciate quantities of the order of magnitude of those that relate to a single particle. To the end of placing such "statistical" inquiries on a firm and certain foundation, Gibbs undertook a general development of the subject. In his own words: "We may imagine a great number of systems of the same nature, but differing in the configurations and velocities which they have at a given instant, and differing not merely infinitesimally, but it may be so as to embrace every conceivable combination of configuration and velocities. And here we may set the problem, not to follow a particular system through its succession of configurations, but to determine how the whole number of systems will be distributed among the various conceivable configurations and velocities at any required time, when the distribution has been given for some one time." Following this programme, the work is primarily an exposition of the methods that must be used in developing this particular branch of the mechanics of conservative systems. The attempt to utilize the results for the establishment of a mechanical theory of thermodynamics is given a distinctly secondary place, and Gibbs utters the wholesome warning that "it should be distinctly stated that, if the results obtained when the numbers of degrees of freedom are enormous coincide sensibly with the general laws of thermodynamics, however interesting and significant this coincidence may be, we are still far from having explained the phenomena of nature with respect to these laws. . . . The phenomena of radiant heat, which certainly should not be neglected in any complete system of thermodynamics, and the electrical phenomena associated with the combination of atoms, seem to show that the hypothesis of systems of a finite number of degrees of freedom is inadequate for the explanation of the properties of bodies."

In the development of his researches, Gibbs worked systemati-

cally and alone. He rarely spoke of a piece of work until it was completed, even refusing at times to say what he was engaged upon at the moment. This was a manifestation of his characteristic unassuming manner and retiring disposition. On one occasion a distinguished French man of science, who had come to New Haven to see Gibbs, called upon him in company with a colleague from one of the language departments who had volunteered to act as interpreter. Gibbs politely accepted the service, and only after a half hour's struggle of the interpreter with unfamiliar technical terms could the embarrassed man bring himself to the admission that he spoke French. Gibbs's lectures were carefully prepared and enriched with a wealth of concrete illustrations, and he held himself always ready to extend help to those who sought it of him, either in person or through correspondence. Academic honors came to him in great number. In 1881 he was the Rumford medalist of the American Academy, and ten years later the Copley medalist of the Royal Society. Four honorary degrees were conferred upon him by American and foreign universities, and he was a member of some fifteen academies and learned societies. A list of his published papers may be found in the American Journal of Science for September, 1903.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE TWENTY-FIRST MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, on Saturday, October 31, 1903, President Webster being in the chair.

The following papers were presented:

1. The Distribution of Motion in a Conducting Gas. P. G. NUTTING.
2. On the Nature of Certain Radiations from the Sulphate of Quinine. FANNY COOK GATES.
3. Phenomena of Imparted Radioactivity. W. J. HAMMER.
4. The Van der Waals *a* in alcohol and ether. EDWIN H. HALL.
5. On the Misuse of Physics by Biologists and Engineers. W. S. FRANKLIN.
6. Mechanical Rotation Produced by the Electrodeless Discharge. (Exhibition of apparatus.) BERGEN DAVIS.
7. Theory of the Electrodeless Discharge. BERGEN DAVIS.

ON THE NATURE OF CERTAIN RADIATIONS FROM THE SULPHATE OF QUININE.[1]

BY FANNY COOK GATES.

WHEN the sulphate of quinine is either heated or cooled through a high range of temperature, it is known to become temporarily phosphorescent and to possess the power of discharging an electroscope.

It has been thought by some that this phenomenon, resulting from simple chemical reactions, is an argument for attributing the radiations of radium and the other active minerals to purely chemical changes. To obtain evidence in favor of or against such a view, experiments were

[1] Abstract of a paper presented at the meeting of the Physical Society held on October 31, 1903.

made ascertaining the nature of the radiations given off by quinine sulphate under the conditions named, and a comparison made between their characteristic properties and those of the radiations from the radioactive substances.

The quinine was sifted uniformly over a metal plate and heated to a known temperature, immediately after which it was placed in a testing vessel between parallel electrodes one of which was joined to the quadrants of an electrometer and the other to the pole of a storage battery, other connections being made as usual. Here it was allowed to cool to a known temperature and the total amount of radiation emitted by the quinine during a given change in temperature was measured by the quantity of electricity discharged through the electrometer. By changing the range and the rate of cooling and the strength and direction of the electric field, curves were obtained showing the exact effect of such variations on the resulting radiation. The absorption of the rays in air and in other substances was also tested.

Unlike the radiations from the radioactive substances, which are incapable of alteration through any change in temperature and appear not to vary with time, the radiations from quinine sulphate are only apparent when accompanied by a great change in temperature. They vary in intensity during this change and in a short time they cease altogether. By increasing the strength of the electric field it is impossible to secure a maximum ionization current from the quinine radiations although a saturation current is readily produced with the radiations from radium, thorium and uranium. The quinine radiations are completely absorbed by a very thin sheet of aluminum and are largely absorbed by two or three millimeters of air and probably by less, which is in marked contrast to even the least penetrating rays of the active elements. An other striking difference in the behavior of the two types of rays is that those from quinine undergo a change in intensity when the direction of the field is changed. When the lower electrode on which the quinine rests is joined to the positive pole of the battery the resulting current is invariably larger than when it is joined to the negative pole.

The variation of the current with electromotive force is very similar to that obtained when ultra-violet light falls on a charged surface except that both positive and negative ions are present. The difficulty in obtaining saturation is doubtless caused by the fact that the ionization takes place very close to the surface of the quinine and that the ions rapidly diffuse to its surface and cannot be removed without a very intense field. The difference in the positive and negative current is probably due to a difference in the size of the positive and negative ions, the positive ions travelling faster than the negative ones; and the difference in size may be explained by the condensation of vapor around them in the same way as the increase in the size of the ions in flame gases when the gases cool.

The effect appears to be due to some type of radiation which is rapidly absorbed in the gas, and since the electrical effects are accompanied by marked phosphoresence it is possible that the ionization is caused by very short waves of ultra-violet light, such as Lenard has shown to be active in ionizing the gas.

These considerations give no evidence for believing that the ionization is due to projected bodies and make it appear certain that the ionizing action of quinine is caused by molecular actions which are influenced by temperature, and not by the spontaneous projection of charged masses from the atom, as in the case of radioactive bodies.

THE THEORY OF THE ELECTRODELESS DISCHARGE.[1]

BY BERGEN DAVIS.

THE theory of the discharge developed is derived from and explains some direct experimental data on the number of volts per centimeter required to produce the electrodeless discharge in gases at various pressures.

The electrodeless discharge was produced by the discharge of a Leyden jar system, which system was charged by the secondary of an induction coil giving 180 discharges per second. The spark gap in the secondary was adjusted until the white discharge just appeared in the vessel.

The volts per centimeter acting around the vessel were directly meas ured by a coil of five turns of wire of a diameter slightly less than that of the vessel. This coil was connected to a micrometer spark gap. The coil that had produced the discharge was lowered over the coil, and the micrometer spark gap adjusted till a spark passed. The potential at this spark gap divided by the length of wire in the measuring coil gave the volts per centimeter acting around the vessel.

Two frequencies of oscillation of the Leyden jar system were used.

Let T_1 and T_2 be the time of a half oscillation for the two frequencies, then

$$T_1 = \tfrac{1}{28} \times 10^{-5} \text{ seconds.}$$
$$T_2 = \tfrac{1}{15} \times 10^{-5} \text{ seconds.}$$

The results obtained were as follows:

1. The volts per centimeter required to produce the discharge depends on the nature of the gas.

2. There is a minimum potential for each gas. The minimum and the pressure at which it occurs depends on the frequency of oscillation.

[1] Abstract of a paper presented at the meeting of the Physical Society, held on October 31, 1903.

3. The volts per centimeter are dependent on the frequency of oscillation for all pressures below the critical pressure, but are independent of the frequency for pressures above the critical point.

4. The number of volts through which an ion must freely run to produce ionization by collision.

5. The ratio of the ionic to the molecular mean free path.

Experiments were made in air, carbon dioxide and helium. The potential gradients obtained for air are shown in curves in Figs. 1 and 2.

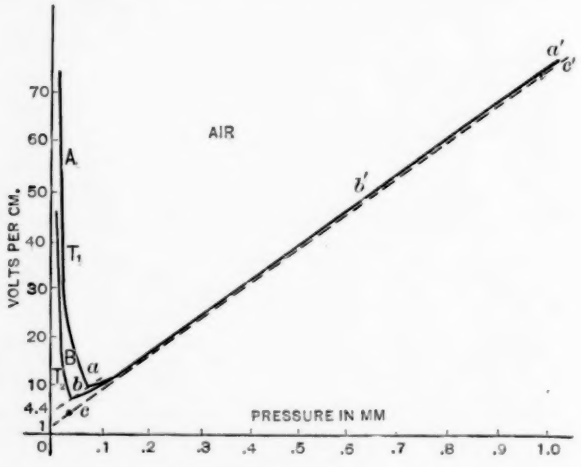

Fig. 1.

Fig. 2 is the part of the curves for pressures below the minimum shown on an enlarged scale.

THEORY OF THE DISCHARGE.

The curves for pressures above the critical are nearly straight lines. They have the equation

$$X = cp + d. \tag{1}$$

The curves of the pressures below the minimum are equilateral hyperbolas, having equation

$$X = \frac{{}_0p_0v}{p}. \tag{2}$$

The ionization is taken as due to impact of other ions. Consider that when the discharge appears, there are N ions present, and before the application of the force there are N_0 ions in a unit volume of gas. Then during a half-oscillation the ionization builds up from N_0 to N ions, according to the equation

$$N = N_0 e^{an}. \tag{3}$$

Where $a = \log b$, and b is the number of ions produced by one ion at each impact including itself. a is the number of successive impacts in T, the time of a half oscillation.

It is assumed that N and N_0 vary in such way that $N/N_0 =$ constant for any pressure, then $aa =$ constant.

The mean path λ of an ion, will be traversed under a force X in time t according to

$$\lambda = \tfrac{1}{2}(X - V_0)\frac{e}{m}t^2 \tag{4}$$

But $a = T/t$, since a is the number of times that the ions go through their free path in time T. Consequently:

$$X = \lambda \frac{2a^2 m}{T^2 e} + v_0. \tag{5}$$

v_0 in equations (4) and (5) represents the total amount of the electrical intensity that is consumed by the delay of the ions after each impact. This quantity v_0 is found to vary inversely as the pressure, that is

$$v_0 = V\lambda.$$

Hence (5) becomes

$$X = \lambda\left(\frac{2a^2 m}{T^2 e} + V\right). \tag{6}$$

Equation (6) is the complete equation for the discharge at pressures below the critical pressure. The constant V is the displacement of the equilateral hyperbola along the axes of coördinates. It is the point of intersection of line (abg) in Fig. 2, with this axis. In air it has the value of 4.4 volts.

The electrical intensity at pressures above the minimum is independent of the frequency of oscillation. The condition to be satisfied is that the electrical intensity X shall be such that $Xe\lambda$ is a constant and equal to w, the energy of ionization by impact.

The equation of the discharge at pressures above the critical pressure is

$$X_m\left[\frac{2}{\pi} + \frac{\pi - 2}{\pi}\sin\frac{\pi}{2}\sqrt{\frac{p - p_0}{p}}\right] = \frac{w}{e\lambda} + d. \tag{7}$$

The left member of this equation represents the departure of the observed curve from a right line. This departure is due to the fact that the applied potential is a harmonic one. At large pressures, the crest of this potential only is effective, while as the critical pressure is approached, the effective force lasts throughout nearly the whole harmonic half-oscillation.

The measuring instrument measured the maximum of this varying potential. Hence

$$X = \frac{2}{\pi} X_m.$$

At pressures above the critical point the controlling condition of the discharge is that $Xe\lambda$ shall attain the value w. At pressures below the minimum (critical) point, the energy $Xe\lambda$ exceeds the value w very much. The controlling condition here is that the ionization must attain the density N for the appearance of the discharge. The critical point occurs at the intersection of these two controlling conditions.

The constants, V and d in equations 6 and 7, are easily determined. The method of finding V has been previously explained. The constant

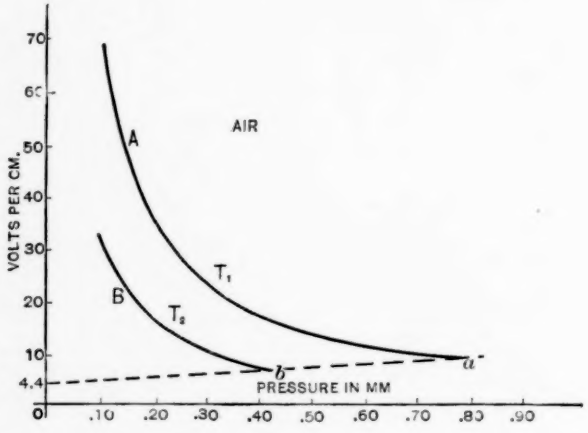

Fig. 2.

d is the intersection of the asymptote to the curve with the axis of coördinates.

The mean free path of an ion at a given pressure may be obtained by solving equation (6) for two values of X corresponding to the two values of T. The two equations so obtained also give the value of a, which is a constant for all pressures and frequencies.

The results obtained for the three gases are given in the following table:

	V	d	λ/λ'	a	P
Air.	4.4	1	4.4	28.5	2.4
CO_2	7.6	2.5	4.3	31.5	1.8
Helium.	8	3.8	3.4	33	1.66

V and d are constants previously explained. λ/λ' is the ratio of the mean free path of an ion to that of a molecule. The constant a is the number of successive impacts which the N_0 ions make during the time of a half oscillation. P is the number of volts through which an ion must run to produce ions by collision.

The ionization is due to the impact of negative ions only. The positive ions, owing to their greater mass, run through only a small part of their mean free path in the time T, while on the other hand the negative ions make a successive impacts in the same time.

My thanks are due to Professor J. J. Thomson for his cordial advice and interest during the progress of this work at the Cavendish Laboratories. I wish also to acknowledge my obligations to the John Tyndall fellowship fund of Columbia University which made this work at the Cavendish Laboratories possible.

NEW BOOKS.

Light Waves and Their Uses. By A. A. MICHELSON. The University of Chicago Press, 1903.

This very welcome addition to our literature in applied optics is based on a series of lectures delivered in the spring of 1899 at the Lowell Institute. After a general statement of wave phenomena and interference, a chapter is devoted to a comparison of the relative efficiency of the microscope, telescope and interferometer for the measurement of small quantities. Then follows a discussion of the application of interference methods to measurements of distances and angles which gives a very clear statement of the methods and arrangement of mirrors best adapted for various purposes. In the lecture on spectroscopic investigations readers will be pleased to find an admirable presentation of the use of the "visibility" curve, a matter not, perhaps, generally understood.

The determination of the length of the standard meter in terms of the wave-length of certain definite radiations is a monument to the skill and patience of the author, and is here quite fully explained. A description of the eschelon grating and a lecture on astronomical applications is followed by a discussion of the ether and an account of experiments made in efforts to determine the relation of the ether to ordinary matter. While investigators in this field are by no means in agreement as to the interpretation of the results, yet the author points out substantial progress toward the solution of this complex problem.

The book is notable for the complete absence of mathematical expressions and for that peculiar attractiveness only found in the writings of actual investigators in their chosen field. No student interested in optical phenomena or their applications should fail to read these lectures with more than ordinary care.

J. S. SHEARER.

Storage Battery Engineering, A Practical Treatise for Engineers. By LAMAR LYNDON. New York, The McGraw Publishing Co., 1903. Pp. 382.

The purpose of this book is to aid engineers in the design and maintenance of storage battery equipments. The book is divided into two nearly equal parts. The first includes the construction and management

of the lead storage battery, while the second part is devoted to a description of the methods in vogue for the application of accumulators to engineering practice.

The theory of the action of accumulators is touched very lightly. But the practical questions of the mechanical design of the cell and the care and management of battery installations are considered in detail. The most striking part of the book, however, is the excellent description of the various methods for using accumulators in connection with distribution and power work. In this connection much attention is given to the consideration of auxiliary apparatus and automatic regulating devices.

ERVIN S. FERRY.

A Calendar of Invention and Discovery. By JOHN CASSAN WAIT. New York, McGraw Publishing Co , 1903.

The subject matter of this calendar is varied and voluminous; scientific and biographical items of all kinds, commonly a dozen or more under one note, have been brought together and arranged in calendar form. The calendar will prove interesting to many, and occasionally valuable, for the miscellaneous contents have been well indexed.

INDEX TO VOLUME XVII.

A.

Air, heat of vaporization of, *Shearer and Strayer*, 124, 469.
Air, velocity and pressure, measurement of, *Zahm*, 410.
Allen, F., Hypotheses of color vision, 151.
Ångström, K., Energy in the visible spectrum of the Hefner standard, 302.
Arrhenius, S., *Lehrbuch der Kosmischen Physik*, 318.

B.

Bancroft, W. D., Whetham's *Theory of Solution*, 192.
Barus, C., Coronal method of estimating atmospheric nucleation, 233.
Baum, F. G., *Alternate Current Transformer*, 317.
Bedell, F., Baum's *Alternate Current Transformer*, 317.
Hooper and Wells' *Electrical Problems*, 128.
Bjerknes, Carl Anton, 125.
Black body, spectral energy curve at room temperature, *Stewart*, 476.
Book Reviews:
Alternate current transformer, *Baum*, 317.
Electrical problems, *Hooper* and *Wells*, 128.
Galvanic batteries, *Bottone*, 64.
Handbook of climatology, *Hann*, 320.
Handbuch der Spectroscopie, *Kayser*, 127.
Lehrbuch der kosmichen Physik, *Arrhenius*, 318.
Lehrbuch der Physik, *Chwolson*, 318.
Light waves and their uses, *Michelson*, 505.
Mathematical papers of the late Geo. Green, *Ferrers*, 319.
Mechanics of machinery, *LeConte*, 316.
Modern chemistry, *Ramsay*, 317.
Principles of inorganic chemistry, *Jones*, 315.
Physical measurements, *Reed and Guthe*, 60.
Storage battery engineering, *Lyndon*, 506.
Physical papers of Henry Augustus Rowland, 62.
Theory of solution, *Whetham*, 192.
Verhandlungen der ersten internationalen seismologischen Konferenz zu Strassburg, *Gerland*, 63.
Vorlesungen ueber Experimentalphysik, *Kundt*, 317.
Vorlesungen über theoretische Physik, von Helmholtz, 191.
Bottone, S. R., *Galvanic Batteries: Their Theory, Construction and Use*, 64.
Buckingham, E., A construction for direct-reading scales for the slide wire bridge, 382.
Note on the deduction of Stefan's law, 277.
On a mechanical method for tracing the curves $x^k y = \text{constant}$, 117.

C.

Chwolson, O. D., *Lehrbuch der Physik*, 318.
Coblentz, W. W., Methods of measuring radiant efficiencies, 267.
Optical properties of iodine, 51.
Coherer, Some further experiments with, *Robinson*, 286.
Color vision, hypotheses of, *Allen*, 151.
Coronal method of estimating atmospheric nucleation, *Barus*, 233.
Cremieu, V., Magnetic effect of electric convection, 385.

Curves $x^ky =$ constant, mechanical method for tracing, *Buckingham*, 117.

D.

Dales, B., Jones' *Principles of Inorganic Chemistry*, 315.
Davis, B., Theory of the electrodeless discharge, 501.
Diffusion in gelatine, *Morse and Pierce*, 129.
Dispersion, rotary, ultra-violet, *Nutting*, 1.
Drew, E. R., Luminous efficiency of vacuum-tube radiation, 321.

E.

Electric convection, magnetic effect of, *Cremieu and Pender*, 385.
Electrifications produced by gases that have been exposed to Roentgen rays, *Zeleny*, 355.
Electrodeless discharge, theory of, *Davis*, 501.
Energy curve, spectral, of a black body at room temperature, *Stewart*, 476.
Energy in the visible spectrum of the Hefner standard, *Ångström*, 302.
Ewell, A. W., Magnetic double refraction, 292.

F.

Falling bodies, southern motion of, *Hall*, 179, 245.
Ferguson, O. J., Quaternions in electrical calculations, 378.
Ferrers, N. M., *Mathematical Papers of the Late George Green*, 319.
Ferry, E. S., Lyndon's *Storage Battery Engineering*, 506.

G.

Galvanometer, *White*, 484.
Gas, conducting, distribution of motion in, *Nutting*, 281.
Gates, F. C., Nature of certain radiations from the sulphate of quinine, 499.
Gelatine, diffusion and supersaturation in, *Morse and Pierce*, 129.
Gerland, G., *Verhandlungen der ersten internationalen seismologischen Konferenz zu Strassburg*, 63.
Gibbs, Josiah Willard, 490.
Green, Geo., *Mathematical Papers*, 319.
Guthe, K. E., *Manual of Physical Measurements*, 60.

H.

Hall, E. H., C_v in liquids and the a of van der Waals in the case of water, 122.
Do falling bodies move south?, 245.
Hann, Julius, *Handbook of Climatology*, 320.
Hartman, L. W., Spectrophotometric study of the luminous radiation from the Nernst lamp glower under varying current density, 65.
Heat of fusion of ice, *Smith*, 193.
Heat of vaporization of air, *Shearer* and *Strayer*, 124, 469.
Heat of vaporization of oxygen, nitrogen and air, *Shearer*, 124, 469.
Hefner standard, energy in the visible spectrum of, *Ångström*, 302.
Helmholtz, H., *Vorlesungen über theoretische Physik*, 191.
Hooper, W. L., *Electrical Problems*, 128.
Hull, G. F., Pressure due to radiation, 26, 91.

I.

Ice, heat of fusion of, *Smith*, 193.
Ingersoll, L. R., Radiant efficiency of the Nernst lamp, 371.
Iodine, optical properties of, *Coblentz*, 51.
Ives, J. E., On the asymmetry of a mercury break, 175.

J.

Jones, H. C., *Principles of Inorganic Chemistry*, 315.

K.

Kayser, H., *Handbuch der Spectroscopie*, 127.
Kundt, A., *Vorlesungen ueber Experimentalphysik*, 317.

L.

LeConte, J., *An Elementary Treatise on the Mechanics of Machinery*, 316.
Lyndon, L., *Storage Battery Engineering*, 506.

M.

Magie, W. F., Specific heat of solutions, 105.
Magnetic double refraction, *Ewell*, 292.
Magnetic effect of electric convection, *Cremieu and Pender*, 385.
Marx, G. H., LeConte's *Elementary Treatise on the Mechanics of Machinery*, 316.
McClellan, W., Thermo-electric behavior of nickel nitrate, 255.
Mercury break, on the asymmetry of, *Ives*, 175.
Merritt, E., Ferrers' *Mathematical Papers of Green*, 319.
Michelson, A. A., *Light Waves and Their Uses*, 506.
Miller, D. C., Reed and Guthe's *Manual of Physical Measurements*, 60.
Morse, H. W., Diffusion and supersaturation in gelatine, 129.
Motion, distribution of, in a conducting gas, *Nutting*, 281.

N.

Nernst lamp glower, spectrophotometric study of, *Hartman*, 65.
Nernst lamp, radiant efficiency of, *Ingersoll*, 371.
Nichols, E. F., Pressure due to radiation, 26, 91.
Nichols, E. L, Arrhenius' *Lehrbuch der Kosmischen Physik*, 318.
Carl Anton Bjerknes, 125.
Chwolson's *Lehrbuch der Physik*, 318.
Kayser's *Handbuch der Spectroscopie*, 127.
Kundt's *Vorlesungen ueber Experimentalphysik*, 317.
Methods of measuring radiant efficiencies, 267.
Physical Papers of Henry Augustus Rowland, 62.
Verhandlungen der ersten internationalen seismologischen Konferenz zu Strassburg, 63.
von Helmholtz's *Vorlesungen über Theoretische Physik*, 191.
Nickel nitrate, thermo-electric behavior of, *McClellan*, 255.
Nitrogen, heat of vaporization of, *Shearer*, 124, 469.
Nucleation, atmospheric, coronal method of estimating, *Barus*, 233.
Nutting, P. G., Distribution of motion in a conducting gas, 281.
Ultra-violet rotary dispersion, 1.

O.

Optical properties of iodine, *Coblentz*, 51.
Oxygen, heat of vaporization of *Shearer*, 469.

P.

Pegram, G. B., Secondary radioactivity in the electrolysis of thorium solutions, 424.
Pender, H., Magnetic effect of electric convection, 385.
Pendulum, compound theory of, *Tatnall*, 460.
Physical Society abstracts, 122, 499.
Minutes, 122, 499.
Pierce, G. W., Diffusion and supersaturation in gelatine, 129.
Pressure due to radiation, *Nichols and Hull*. 26, 91.

Q.

Quaternions in electrical calculations, *Ferguson*, 378.
Quinine, certain radiations from, *Gates*, 499.

R.

Radiant efficiencies, methods of measuring, *Nichols and Coblentz*, 267.
Radiant efficiency of the Nernst lamp, *Ingersoll*, 371.
Radiation, pressure due to, *Nichols and Hull*, 26, 91.
Radiation, spectrophotometric study of, from the Nernst lamp glower, *Hartman*, 65.
Radiation, undescribed form of, *Sanford*, 441.
Radiation, vacuum-tube, luminous efficiency of *Drew*, 321.
Radiations, from the sulphate of quinine, *Gates*, 499.
Radioactivity, secondary, in the electrolysis of thorium solutions, *Pegram*, 424.

Ramsay, W., *Modern Chemistry*, 317.
Reed, J. O., *Manual of Physical Measurements*, 60.
Refraction, magnetic, *Ewell*, 292.
Resistances, low, method for the comparison of, *Wenner*, 384.
Robinson, P. E., Some further experiments with the coherer, 286.
Roentgen rays, electrifications produced by gases that have been exposed to, *Zeleny*, 355.
Rowland, H. A., *Physical Papers*, 62.

S.

Sanford, F., On an undescribed form of radiation, 441.
Scales for the slide wire bridge, *Buckingham*, 382.
Shearer, J. S., Heat of vaporization of air as related to its composition, 124.
Heat of vaporization of oxygen, nitrogen and air, 469.
Heat of vaporization of nitrogen, 124.
Michelson's *Light Waves and Their Uses*, 506.
Sheldon, S., Bottone's *Galvanic Batteries*, 64.
Slide wire bridge, construction of direct-reading scales for, *Buckingham*, 382.
Smith, A. W , A determination of the heat of fusion of ice, 193.
Specific heat of solutions, *Magie*, 105.
Spectrum of the Hefner standard, energy in, *Ångström*, 302.
Stefan's law, deduction of, *Buckingham*, 277.
Stewart, G. W., Spectral energy curve of a black body at room temperature, 476.
Strayer, F. R., Heat of vaporization of air as related to its composition, 124.
Supersaturation, in gelatine, *Morse and Pierce*, 129.

T.

Tarr, R. S., Hann's *Handbook of Climatology*, 320.
Tatnall, R. R., Theory of the compound pendulum, 460.
Thermo-electric behavior of nickel nitrate, *McClellan*, 255.
Thorium solutions, secondary radioactivity in the electrolysis of, *Pegram*, 424.
Trevor, J. E., *Josiah Willard Gibbs*, 490.

U.

Ultra-violet rotary dispersion, *Nutting*, 1.

V.

Vacuum-tube radiation, luminous efficiency of, *Drew*, 321.

W.

Wells, R. T., *Electrical Problems*, 128.
Wenner, F., A method for the comparison of low resistances, 384.
Whetham, W. C. D., *Treatise on the Theory of Solution Including the Phenomena of Electrolysis*, 192.
White, W. P., A convenient galvanometer, 484.
Whittelsey, T., Ramsay's *Modern Chemistry*, 317.

Z.

Zahm, A. F., Measurement of air velocity and pressure, 410.
Zeleny, J., Electrifications produced by gases that have been exposed to Roentgen rays, 355.

JAN 19 1917